'Creating Measures' Crowded-ness
Task - Example #4 (solutions)

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This problem gives you the chance to:
  • criticise a given measure for the concept of "crowded-ness"
  • invent your own way of measuring this concept and use it effectively

____________________________________________________

<table>
<thead>
<tr>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Group A" /></td>
<td><img src="image2.png" alt="Group B" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group C</th>
<th>Group D</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Group C" /></td>
<td><img src="image4.png" alt="Group D" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group E</th>
<th>Group F</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5.png" alt="Group E" /></td>
<td><img src="image6.png" alt="Group F" /></td>
</tr>
</tbody>
</table>
Warm-up

In your opinion, which group looks the least "crowded"?
Which looks the most "crowded"?
Put the groups in order of "crowdedness", just by looking.

Comment:
Intuitively, most people would consider the least crowded to be F, and the most crowded group to be C and, they would give the following rough order for "crowdedness": F, B, A, C. The remaining groups are quite 'odd' in that D seems to be distributed less evenly and E seems to be distributed somewhat linearly. There should therefore be some disagreement about these points. Intuitively, "crowdedness" gives the feeling of "closeness" or "confined-ness"; thus it may mean the proximity of one person to another or the denseness of a collection of people in a defined space.

1. Someone has suggested that "crowdedness" may be defined as the area of a box that will just enclose all the people in a cluster. So for group A:

   ![Diagram of Group A]

   "Crowded-ness" = 7.26 m²

   Use this method to calculate the crowdedness of group F:

   ![Diagram of Group F]

   Solution:
   The solution for group F gives a rectangle which is approximately 27 m² in area.

2. Write down at least two reasons why this is not a good way of measuring "crowdedness".

   Solution:
   This area is not a good measure for "crowdedness," because it takes no account of the number of people in the rectangle. Also, there is more than one rectangle that just touches the extreme people, giving a different area. For example:
3. Describe a better method for measuring "crowdedness". Explain why it is better.

**Solution:**
A better solution would be one in which we
- provide an unambiguous method for defining the area occupied.
- divide the number of people by this area, to give a 'density' (people per unit area) definition.

For example, we could define "crowdedness" as:

a) The number of people divided by the area of the smallest (convex) polygon that can contain the set of people.

b) The number of people divided by the area of the smallest circle that will surround the people.

(Other possible measures might use distances between people, for instance: the distance between the two most remote people, or the sum of the distances between pairs of people.)

4. Use your method to give a numerical value to the "crowdedness" of Groups A to F. Use your method to put the groups in order of "crowdedness".

**Solution:**
Next, having chosen a measure of compactness, the student must show its application by computing numerical compactness values for the given groups.
Using the definition: area of smallest circle that will surround the group divided by the number of people within it, I obtained the following results (units are in people per square meter).

<table>
<thead>
<tr>
<th>Group</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crowded-ness</td>
<td>1.06</td>
<td>0.80</td>
<td>1.59</td>
<td>0.45</td>
<td>0.64</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Thus the order from the least crowded to most crowded, using this definition is:  
F, D, E, B, A, C.