This problem gives you the chance to:

- criticise a given measure for the concept of "steep-ness"
- invent your own ways of measuring this concept
- examine the advantages and disadvantages of different methods.

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Warm-up

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'Creating Measures' Steep-ness
Task - Example #2 (solutions)
Without measuring anything, put the above staircases in order of "steep-ness."

**Comment:**
This first question is simply intended to orientate the students to the task. It may be used as a class introduction.

1. Someone has suggested that a good measure of "steep-ness" is to calculate the difference:

   **Height of step - length of step**

   for each staircase. Use this definition to put the staircases in order of "steep-ness." Show all your work.

   **Solution**
   Using the measure 'height of each step - length of each step', the 'steep-ness' of each staircase is given in the table below (using centimeters as the unit).

<table>
<thead>
<tr>
<th>Staircase</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>1.5</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>Length (cm)</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3.33</td>
</tr>
<tr>
<td>Height-Length (cm)</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
<td>-1</td>
<td>-2.08</td>
</tr>
</tbody>
</table>

   Using this measure, the staircases in order from most to least steep are: D, A and B and C (tie), E, F.

2. Using your results, give reasons why Height of step - length of step is not a suitable measure for "steep-ness."

   **Solution:**
   The above measure is unsatisfactory because:
   
   - It gives no real indication of the steepness. Using this measure, A and C are labeled as equally steep, which does not fit with intuition.

   - It is dependent on the units used. If we use inches instead of centimetres we get a different "steep-ness" measure.

   - It is usually negative, which is inelegant and awkward to use.

3. Invent a better way of measuring "steep-ness." Describe your method carefully below:

   **Solution:**
There are many other ways of measuring "steep-ness." Students might, for example, propose using:
   a) The angle of inclination;
   b) The ratio of 'step height'/ 'step length' (technically: riser/run);
   c) The ratio of 'height of whole staircase'/ 'length of whole staircase';

These are equally sensible, and equivalent, except is may be sometimes unclear what we measure as the 'length' of the staircase.

4. Place the staircases in order of "steep-ness" using your method. Show all your work.

   Solution:
   Whichever measure we now use (a), (b) or (c), we obtain the same order for the staircases.

<table>
<thead>
<tr>
<th>Staircase</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>1.5</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>Length (cm)</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3.33</td>
</tr>
<tr>
<td>Height (2 d.p.)</td>
<td>0.75 (3/4)</td>
<td>0.67 (2/3)</td>
<td>0.5 (1/2)</td>
<td>1 (1/1)</td>
<td>0.67 (2/3)</td>
<td>0.38 (3/8)</td>
</tr>
<tr>
<td>Angle of inclination (nearest degree)</td>
<td>37º</td>
<td>34º</td>
<td>27º</td>
<td>45º</td>
<td>34º</td>
<td>21º</td>
</tr>
</tbody>
</table>

This gives the order of steep-ness (from most to least steep) as:
   **D, A, B and E (tie), C and F.**

5. Do you think your measure is a good way of measuring "steep-ness?" Explain your reasoning carefully.

   Solution:
   Here we would like students to review their results critically and decide whether the results from their measurements accord with their intuitions.

6. Describe a different way of measuring "steep-ness."
   Compare the two methods you invented. Which is best? Why?

   Solution:
   This question provides an opportunity for students to look for an alternative measure.