Math 141
Evaluation Report

Spring 1997

September 22, 1997

prepared
for

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by
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This work was supported by a grant from the UW-Madison Assessment Council.

1. INTRODUCTION

This report details the backgrounds and experiences of students in Professor Philip E. Miles' section of Math 141 which was offered spring semester 1997. Math 141 is designed as an alternative to Math 112 in fulfillment of the first component of the university's general education requirement in quantitative reasoning (QR-A).[1] Generally speaking, Math 141 is aimed at students who do not plan mathematics or mathematics-intensive majors; its general goal is that students will learn to think "mathematically" and become independent problem solvers.

This report, developed and presented by the LEAD (Learning through Evaluation, Adaptation and Dissemination) Center on the University of Wisconsin-Madison campus, is based on classroom observations and, in particular, on the inductive analysis of exploratory, semi-structured interviews with 9 of the 22 students originally enrolled in the section. Interviews were conducted by two LEAD Center evaluators, Phil Miller and Ramona Gunter, during the period of April 17 through May 5, 1997.

Our approach is student-focused; we are interested in what kinds of teaching strategies and techniques seemed most effective from a student point of view. The data in this report are intended to be used along with quantitative information (such as test scores or conventional student evaluation instruments) that the department and others concerned with QR-A planning have collected, and it should not be considered a definitive or summative statement about the course in and of itself. Rather, LEAD considers this "formative feedback" information designed to help the instructor, the department, and the university "fine tune" the QR-A program in response to student needs and perceptions and within the context of program objectives.

In the following paragraphs interview excerpts from the students in this particular section of Math 141 are shown in italics. Where there is no identification for the quote, it is the comment of a student interviewee. Where there is dialogue, "I" signifies interviewer and "R" signifies respondent. Ellipses [(…)] in quoted material indicate deleted dialogue occurring within the reproduced material. Ellipses are used so that the reader can appreciate the speaker's views on a particular issue without having to sort through parts of the raw dialogue which are not relevant or informative. A row of asterisks indicates material from a different interview.

2. THE STUDENTS

2.1. Previous Math Experiences/ "Readiness"

All students enrolled in this Math 141 section began the course with the minimum credit hours in high school math mandated for entrance to the university. The prerequisite for this section was placement into Math 112 (college algebra), though a few exceptions were allowed to enroll. On the basis of students' own self-reports about performance in high school math, the majority in this

[1] Several discipline-specific alternatives are available to fulfill the second half of the requirement (the QR-B).

section performed adequately; many[2] stated that they had actually done very well in those classes. We acknowledge that the meaning of grades varies among schools. These self-reports are not intended to suggest that the students were homogeneous in their levels of math skills, nor that they had mastered the material in high school math courses. But they do suggest, and the students' acceptance to UW-Madison confirms, that the students were capable of developing skills and strategies helping them to get through typical high school math courses.

Most of the students interviewed, including some who reported strong performances in high school math courses, admitted that they had always had to work hard for whatever grades they received. One student confided: "I had to work on [math] where other subjects seemed a lot easier." Another student described his/her struggle with the subject matter this way:

> *I've always struggled through it in high school. I've always worked really hard to try and do well in math. But it's always been a really hard thing for me. (…) I placed into [Math] 101 so I took that last semester and I got a B in that class. But I had to work really hard.*

The diversity of students' math experiences appeared to be broad. While a few students admitted that they had not even satisfied the prerequisite of readiness for Math 112, a few others had already taken Math 112 or a higher level college math class. The following student talked about how s/he came to realize others had more math background:

> *I can remember there were a couple of these initial problems[when] I overheard people say: "This is easy, I learned this in--" I think somebody said they were in 112. (…) He just said: "Oh, I learned this in 112. You just take the cube root and da da da." I'm like: "Cube root? What's a cube root?" You know?*

The above student noted that some students were more prepared than others to start Math 141. However, it is not apparent that being less prepared than his/her classmates hindered him/her in any way. The student went on to say: "This is way back in the beginning [when I didn't understand]. Now I know what a cube root is. Now I know how to apply that."

2.2. Attitudes Toward Math

- **Affective outcomes dependent on learning or performance outcomes**

A few students expressed a certain satisfaction found in success with math, but frustration over "not getting it." The following comment represents the point of view of these students:

> *I like [math] if I can get it. (…) I love it, you know, if I can work it out -- it's great. But if I can't figure it out, and not understand what the teacher's saying, then it's very frustrating.*

- **Math is harder than other subjects**

Not surprisingly, most of the students in this section expressed that, for themselves, math requires a greater level of effort than other subjects. As one student said: "[Math 141 is] definitely one of my harder classes; but math has always been. (…) It's just that math in general is always harder

[2] Qualifiers are used to give indication of how many students expressed any given viewpoint: "few" indicating 1-3 students; "many" indicating 4-7; and "most" indicating 8-9.

for me than anything else." The following student talked about working hard for grades in classes that other students thought were easy:

> *I usually would end up with a B [in my high school math classes]. Maybe in algebra I'd have to work for the B where the other students [thought it was] pretty easy.*

- **Math is intimidating**

One aspect of math which many students described as intimidating was that it is "unforgiving." That is, students described the process of doing math as being "right" or "wrong." Therefore, being slightly off will result in having done the problem wrong. Because of this, some described feeling vulnerable and apprehensive about doing math. Intensifying the situation is that at every step in the problem-solving process, the student must make a decision which will be "right" or "wrong." One student expressed his/her apprehensions in the following way:

> *Well, I think part of my problem is that with math -- (…) we're doing things like a number set, and if you miss one, you mess up the whole problem. That type of thing. So, that really scares me.*

- **Disinterest in/dislike of math**

Many students were explicit in their dislike for or disinterest in math as a subject area. One student expressed his/her dislike for math, saying: "Thankfully there are careers that aren't focused around it." In spite of, or perhaps because of a lot of exposure to math, the following student also established that s/he does not like it:

> *Well I never really liked math {laughs}. I'm not really math-oriented myself. I took algebra, a year of geometry, a second year of algebra and trig, or trig/algebra II/ trig. And then I took a semester of pre-calc in high school.*

3. WHY STUDENTS ENROLLED IN MATH 141

Students expressed a few different motivations for why they registered for Math 141.

- **Avoid alternatives**

Most of the students made some reference to having heard that Math 112 (the alternative to Math 141 in the satisfaction of the QR-A) was more difficult than Math 141. The following is representative:

> *R: After I heard how many people were having so much difficulty with Math 112, I decided -- because I remember what my advisor had said -- so I signed up for Math 141.*

I: And what did your advisor tell you about the class?

R: She told me that this would fill the same requirement that Math 112 did. So my roommate was in Math 141 last semester and (…) I didn't hear her complain about it. [And I had] heard people complaining about Math 112. (…) She didn't fail [Math 141] like other people in Math 112.

(…)

I: Okay. So basically, you took it because you thought it was different from 112. You thought it was easier than 112. You thought it was --

R: I guess I thought it was easier. I thought that I would receive a better grade than I would in 112.

In fact, many students expressed that they did not know anything about Math 141, and in some cases, were unable to find information. One student said, "…no one knew anything about it. I asked people, 'Do you know anything about Math 141?' and they said, 'No'." Because Math 141 is a new course, it is not surprising that students knew more about the alternative courses they wished to avoid.

- **Interest in course content**

While avoiding Math 112 was a major reason for taking Math 141, a few students also expressed an interest in the course content:

I did not want to take 112, because I'd just heard a lot of things about it being really difficult. And 141 sounded like it wasn't really for people who were going to major in math. So I thought I should give it a good try. And they said that it was going to apply a little more to real life.

- **Satisfy requirement**

A few, however, made it clear that they were not interested in course content. When one student was asked further if s/he might also have wanted to learn something with respect to math, s/he responded: "That would be helpful. However, I wasn't going into the class thinking that. I wouldn't have taken it if I didn't have to." In short, these students probably would not have signed up for Math 141 (or any other math class) if they had not been required to do so. The following student exemplifies this sentiment:

R: [I took Math 141] because I had to take math. That would be the main reason. I wouldn't have taken anything if I didn't have to.

I: So you wouldn't have taken it just to learn?

R: No. If so, I would have taken something more interesting to me.

4. ASPECTS OF THE COURSE AND RESULTING FINDINGS

In general, students did not come to Math 141 with necessary skills to engage in independent problem-solving. Rather, they described their previous math experiences as using problem-sets to practice application of explicitly defined algorithms. This view is exemplified by the following student, who indicated that s/he is more comfortable doing math by relying on algorithms:

I: What kinds of examples are most helpful for you?

R: Usually the formulas, because I like to have "this stands for this and this stands for this." If I can have one way to solve a bunch of different problems, then I can usually make the transfer between the problems. If it's just a bunch of words and numbers that don't go together except in this one particular case, then I kind of get lost when it comes to a new case.

4.1. Content and Presentation

- **Value of word problems**

As stated in section 3, students did not know very much about Math 141 other than that it was not a typical algebra course. Some knew the focus would be on solving word problems. Many commented that they liked this focus. The following statements are representative:

The thing I like about this class is that it's word problems that we're doing -- and there's not a specific way you have to do it. There are a bunch of ways that you can go about solving [problems] -- which I really like.

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[Using word problems] is always something I've been better at. Maybe I feel a little more comfortable because there're words in it. Because it's not just straight numbers.

For students who were looking to avoid another traditional algebra class (routine solutions and "straight numbers"), word problems seemed to be the right content for Math 141. However, the reasons for which many preferred word problems were the very same reasons for which a few students disliked them. The following student described a sense of confusion regarding how to solve word problems, and subsequently, how to ask for help on them:

It's hard to get help on the problems because it's not really a concrete kind of math. It's not like [how] you ask for help in calc or trig. It's a weird kind of math. You know, story problem kind of things.

- **Scope of word problems**

While most students have studied word problems as a formal part of their high school math courses, many noted that they have never had a course which focused solely on them. The variety of topics addressed by Math 141 word problems impressed many. A few specifically noted the utility of some of what they were learning, not only in their everyday lives, but in the remainder of the course as well. The following student talked about the variety of problems covered and said the course content had provided him/her with some tools which could be applied later in the course.

[We're learning] a variety of different things. From how many pennies fit in a cube to distance-time problems, or percent-change problems. A variety of different things. Word problems and stuff like that. (…)In the beginning, we had a bunch of problems on numbers, like how many [of these objects] fit [in a certain volume], and that kind of thing, and you just solved it. There's a certain way to do it that made it

all easy and -- a certain equation that you wouldn't think you would be able to use by looking at the problem. And we did estimation problems which helps us in the long run for everything. We did them in the beginning.

4.2. Materials

- **Packet and answer sheets**

The lack of a text in this course was, for most students, largely off-set by the supplemental materials provided. Favorable comments about both the packet and the solution sheets follow:

When I first got the packet I thought: "How is this forty-page packet going to occupy us for the whole semester?" And we're on the last two pages, and we've got--what? three weeks left? But, I mean it's thorough. And a lot of it's nice, too.

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I'd have to say [that] the solution sheets that he hands out after the projects [have been very beneficial]. Because I can actually look at it on my own time and figure it out.

Although many students liked the course packet, a few felt that a textbook might provide some extra structure or guidance:

I: Do you like the (….) workbook that he's got? And the projects that he asks you to do -- do you like the materials that he uses in the class?

R: I think if maybe there was a little bit more of an actual text to it, it would add some structure in the class. (…) I know last semester they had, I forget what the textbook was like. But they did have an actual textbook that they followed. I don't know what was involved in it, but I know they actually had a textbook.

I: And that would provide some of the structure that you're looking for?

R: Maybe, yeah. Maybe you would understand exactly what was going on in class. You could read and kind of have it reiterated to you by reading a textbook. You hear just from one example that he gives, like the one project is the only example of the concept he's teaching. So if you don't understand that one problem, you have nothing else to back it up with.

- **Examples and answers to problem sets**

Although the following student does not specifically state that s/he wants a textbook for the class, what she <u>does</u> want reflects what is lacking in absence of a text. In the following dialogue, this student stated that s/he wants more examples, and answers to homework problems:

I: You [said you aren't] as successful as you'd like to be [in this class]. What would you like to be different about the course? It sounds to me like you liked the idea of this course --

R: I really do. I really do like the idea. Right. I've been saying, "Examples, examples." I think that would be helpful -- I really need to have more examples.

I: Have you asked for more examples from him personally?

R: M-hm, And he tries, he does try. I mean he really does try. There have been some good examples and some examples I don't understand. But definitely [I want] more examples. And maybe even answers in the back of the packet.

A text provides an outline or structured content that can give students a sense of security. It also can offer examples and problem sets that give students opportunity to practice and to recognize patterns in the problem-solving process that will help students generalize what they are learning. The following student articulated this, and echoing the feelings of the student above:

I: (…) so what kinds of things would make [this class] even better, a better experience for you?

R: I think a textbook, or not even a textbook but something that had a lot of problems. Like [when we do] a project in class -- have the next page be 15 problems. If a project's three problems, have the next page be 15 problems of each of those kinds to practice them -- so people could practice them. Because [the problems are] going to come up on the exam and they're not [going to be] the same. And have slight variations in them -- just don't pop it on people in the test. And, for me, it would have been beneficial had he gone over a problem on the board and then we [work on] the exact same thing basically, the same problem with the same variables, just different numbers basically, in class. And then move on to a slight variation from there. I think that would help.

- **Learning through doing vs. learning through reading**

Another student, however, shared an alternate perspective about the lack of a text book. This student does not address the concerns of the students above, however. (In fact, with respect to problem sets, this student described the problems in the homework packets as being equivalent to much larger, but easier, problem sets found in texts.) Instead, their focus is on the idea of learning this problem-solving process through doing rather than reading about it. In the following dialogue, this student explained that they didn't think they could learn this process through reading:

I: What seems to be making it easier for you to learn?

R: Well, I think for one, not having the structure of the textbook makes it a lot easier. I haven't seen the 112 textbook, so I can't tell you. But from my past experience in math -- Isn't there usually like a chapter where you have to read something? Usually? I mean --

I: Yes. You know, it depends on the course, too. But in a lot of books there's a text and then a problem set that goes with the text.

R: Yeah, right. So, I mean, I couldn't even imagine trying to understand what somebody's written about some of the stuff we've been doing. And then also the fact of having fifty problems at the end of the chapter. You know? Just, to me, seems so uninteresting. Where we have -- we still have a half a dozen problems for each section that we're working on, but each problem is probably, you know, equivalent to about ten of those [textbook "equation-like"] problems.

4.3. Assessment

Most students felt that exams and quizzes provided good assessments of what they were learning. The following are representative:

> *R: He gave us the perfect review sheet [for the exam] last time.*
>
> *I: And it sounds like the kinds of problems you're solving in class are the kinds of problems he asks you on the exams?*
>
> *R: Yeah.*
>
> *I: So no surprises?*
>
> *R: Not too bad. I mean they're harder but --*
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> **************
>
> *R: We do these projects once a week, and then we have a quiz on Friday. We do projects in groups, and then we're quizzed on the concepts. That's pretty much how the class goes.*
>
> *I: Do you think the quizzes are a good assessment of the kinds of things you've learned?*
>
> *R: Yeah. [Although] the first week, he actually scared us pretty good. Because the first week, the project we did was REALLY hard. And (…) I ended up getting it, but it took me two hours to do. (…) And I was just like – "Oh my god," And then the quiz was very similar. Yet we only had like ten minutes to do it, and -- I mean I did okay. But it was just like—"oh oh," you know? It was scary. But it turned out that [the course is] definitely do-able. Especially with the groups, it's nice.*
>
> *I: So you think that his projects got easier, or you just--*
>
> *R: I don't know if it actually got easier or if it's just that we got used to, that's how it was going to be, you know?*

However, a few students felt the exams were not a fair assessment. The following student focused on the difference between how solutions for homework and exam problems were to be presented:

> *I: Do you think the exams measure what you're learning in class very well?*
>
> *R: No, I don't think so. Because everything in class we have to do, we have to write. We have to explain everything step by step and write [about] everything. Everything we hand in, there's like two, three pages typed of every project we hand in. And on the exams, you're not supposed to write anything. (…) I mean, there are linkages between the problems, but the way in which we're supposed to express how we answer them is completely different. I guess I can understand, because (…)on quizzes and exams you're not supposed to use words; you're supposed to do the work. Whereas on other things you're supposed to go through it all.*

4.4. Pedagogy

- **Question-Answer**

Many students recognized and were able to articulate, at least to some degree, the professor's teaching strategy. (See also section 4.5.) The relationship between students' questions and

professor's answers was a prevailing theme; that is, the students' questions are never directly answered. Students identified the goal of this strategy as that of getting students to figure problems out for themselves. This aspect of the course made it stand out from other math classes. One student said Math 141 is "actually really different. A lot of times you'll ask teachers questions and they'll hint to you what you're supposed to be doing. (…) He will not do that. (…) He wants us to get it on our own." This student continued:

> *I know that people in that class do have [problems figuring things out on their own and] get really frustrated. (…) [They get frustrated by] the way [the professor] handles them when he doesn't tell them anything (…) I see how upset they get. (…) I think that he's pretty much doing the right thing. (…) I mean, he just wants you to figure it out on your own. And it can be done.*

In this class, students solve problems. The following comment is representative of other students' descriptions:

> *When he lectures and when he walks around and listens to us, he won't give us the answers. He wants us to figure the problems out. And (…)in previous math classes they would give you the answer right away, but he won't. He won't give them to us. He really wants us to figure it out. So I think that's a little different. (…) It's different, too, because he doesn't lecture for a long time. He does lecture, but it's more group work. It's us working together.*

The way in which the professor responds to student questions is a source of frustration:

> *[Professor Miles] wanders around the room. [Someone will say] "Professor Miles, I have a question," and then he'll come over. [One time our group] had a question, and he came over and [we asked him the question] and he said, "That's a valid point. What do you think you should do about it?" At the time, it's kind of frustrating. But I guess his point is that he doesn't want to give anything away. So he said we should draw a diagram. That was his suggestion.*
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> ***************
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> *I: You don't feel like he answers specifically, "This is how you do this"?*
>
> *R: Sometimes he might. However, many times he goes into, "This is how you're <u>not</u> supposed to do it." I feel that's how [he does it].*
>
> *I: And that kind of explanation doesn't help you.*
>
> *R: It helps, because I know what <u>not</u> to do. But if we're having a quiz on Friday and we end the class saying what <u>not</u> to do, I still don't know what <u>TO</u> do.*
>
> *I: So you want something a little different than that.*
>
> *R: Yeah. I mean, I think that it would be helpful after giving us how to do it, or you know, one of the ways to do it and how not to do it as well so that we know, but I think that the way <u>TO</u> do it is more important than the way <u>NOT</u> to do it.*

The following student described the professor's pedagogy, intended to lead students through the problem-solving process:

I: How do you feel about the way the professor answers questions? Are [the answers] helpful?

R: He'll answer a question with a question. He'll always say, like, "Well, hmmm." And he'll always ask another question; so it's kind of back to square one then.

I: Do most of those questions he asks get answered?

R: Eventually. Not right away. But he keeps asking more and more questions until finally someone will say the right answer.

I: So do you feel that you actually learn from that process? Are you able to figure out what he's trying to get at?

R: Yeah. (…) [The professor] never says, "Yes you're right," or "No, you're wrong." He always leads us up to a conclusion on our own.

For many of these students, a math class is a frustrating experience. This particular aspect of the professor's teaching style was one of the more frequently mentioned causes of frustration. While many students mentioned its effectiveness as a teaching strategy – they admitted so grudgingly. One student said that while s/he would prefer that the professor simply answer their questions, it was good that he did not. In other words, the teaching strategy was like medicine that was good for you, but unpleasant to take. While admitting that this type of question-answer was an aspect of the course s/he was learning from, the following student said s/he did not necessarily appreciate it:

I think that when we ask questions, they could be answered a little more concretely instead of challenged. Even if our answers are right, he'll still challenge what we say. He'll try to throw us off sometimes or try to get us to think about other sides of the problem. I think it would be more helpful if he just gave us a, "Yes, you're right," or "No, you're not right," instead of not really letting us know.

Evaluators' Viewpoint:
In the two excerpts above, students reveal some competing understandings of, or ambiguous feelings regarding, their learning experiences. In the first excerpt the student attributes to the professor's responses both of the following: leading them back to "square one" and leading them up to the conclusion. In the second excerpt the professor is credited with trying to throw them off track and trying to get them to think about the problem from other sides. These ambiguous interpretations of the professor's intentions suggest that some students felt less than confident (at least with some aspects) of the learning process in which they were engaged.

- **Use of writing**

Another component of the course structure which engendered cognitive involvement on the part of the students was the writing assignments. Many students mentioned writing as a routine part of their projects. A few elaborated that writing helped them to think about the math they were doing. The following student described the benefits of the writing:

R: It has been somewhat beneficial to have to write my answers out in paragraph form, because it makes me break things down in my own head.

I: Have you ever done that before? Have you ever thought about math [that way]?

R: No, that's probably why I never did well in math -- because I can't think of it from an analytical [perspective].

- **Implicit agenda**

The explicit agenda of Math 141 is to utilize critical thinking and mathematical reasoning to become independent problem solvers. Implicitly, students will learn and use applications for various concepts and skills in algebra. At least one student noted that this implicit agenda "snuck up" on him/her. That is, this student wasn't aware of the level of algebra s/he learned and used in Math 141 until it was pointed out. As well, this student stated that if s/he had been aware up-front, s/he would have been intimidated. The following exchange, beginning with a description of how a teacher in an earlier course mitigated a student's fears, illustrates the circumstances of lessened apprehension:

R: [The teacher] had a lot of hand-outs. And he wouldn't say, you know: "Today, we're going to learn the Pythagorean theorem." He would show us the different measurements that go into it, and the different angles that you use, and all this other stuff. And then he would say: "Oh, and by the way, you just learned the Pythagorean theorem." You know what I mean?

I: So he kind of sneaked it in?

R: Yeah, exactly, exactly. So that was a really good technique, I thought. And I think I noticed that in some of the stuff in 141. (…) The way they teach [some of these problems] in 141, it's kind of the same way, it's an indirect approach. And then I went in to talk to Professor Miles. He showed me in the college algebra textbook that all along we've been learning these quadratic equations-- which gosh-- if I would have seen that in the beginning of the chapter, I would have been like: "Psh. I can't learn this!" You know what I mean? It was like I have myself convinced that I can't comprehend this stuff, I think. Because I think I have this, I don't want to say fear, but like apprehension about it.

4.5. Class structure

Students described the class structure as one where the professor spends a relatively short time presenting new material. For the remaining period, students work in groups on challenging problems.

- **Cooperative groups**

Group work was a defining characteristic of this course. When asked how s/he would describe Math 141 to a friend, one student stated:

Well, I would say that the one nice thing about 141 is that you're working in groups. So you're not always on your own. You have other group members to talk through the problems. I would say that it's not like any other math class I've ever taken.

Most students liked the group work. The following explained that the process of solving problems with a group is a good learning experience:

We're working in groups. And the last project was [worth] double points. (…) It was called a ping-pong project; it was fitting the number of ping-pong balls in the room, including all of the different odd shapes. (...) And we really beat that thing to death. I mean we took it apart, every little nook and cranny. (…) It was like a brainstorming thing, you know? Which I really, truly believe is a really good way to learn.

Many found that discovering the perspectives of their classmates through group interaction was a valuable component in the learning process:

I like the group work a lot. Because sometimes someone will just say something -- a lot of times we'll just throw things out until we come up with the right thing. It seems to work pretty good. It takes a lot of time, but it works good.

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That's what I like about working in groups, is that each person might be seeing something different.

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I: You said the group works well. I'm wondering what it is about the groups that seem to be working?

R: I don't know. Just everyone works together, and everyone adds a little bit of information and we all work together and solve the problem -- it's helpful.

I: So there's never just one person who says, "Hey I know how to do this," and just --

R: Sure, sometimes there is. But then it's helpful for everyone else to see how they did that and to see if their way would work as well.

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I: What works about [the group approach to problem-solving]?

R: Well, just the fact that we can pool our ideas together. If there's something you don't understand, someone there can explain it.

Indeed, for some members of the class, group work was considered critical to their success:

I think that each student in the class grasps a little bit of what [the professor is] trying to say. It's very hard though. But once we put all of the pieces together -- I feel that if a student asks him a personalized question, they have a question in their mind that they want to figure out. Sometimes he'll answer that, but he won't necessarily know why they want to know it. So once they figure that out, it's helpful when we go into a group and have trouble understanding something, it'll help to bring that information back to the group. (…) I think that the group work is great. I don't know what I would do in the class without it.

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I: Does it help [to figure out the problems] when you're working in a group?

R: Oh, definitely. I would say that I would not be able to survive in that class if I wasn't working in groups.

- **Students as resources**

According to students, everyone participates in the problem-solving process. Their descriptions of how they solve problems in class suggests that they see themselves as their only resource. As one student said: "[The group work is] good because you're helping each other. Because, like I said, I'm not getting that help from the professor -- personally I'm not. So I'm getting help from other students. We're kind of helping each other." Of note is that students do not appear to see the professor as a resource (many saying that he never answers their questions). As one described it: "He doesn't always answer you straightforward and so you're sitting there just dumbfounded,

saying, ‘I asked you a question, but you didn’t really answer it at all’.” The following excerpt reinforces our interpretation that students focused only (or at least primarily) on themselves as resources:

> *Most of the time we split up into groups. And then sometimes we'll get back together as a whole and one of the groups will be picked to show you how they went about trying to solve the problem -- so we have some way of knowing if ours was right or not.*

The following student talked about what happened when the group got “stuck”:

> *I: And what's the process that your group goes through when it's trying to solve the problem?*
>
> *R: We just think of everything possible that could come out to get the right answer.*
>
> *I: OK, and everybody is talking?*
>
> *R: Yeah, everyone contributes.*
>
> *I: And when you get stuck, do you have any resources?*
>
> *R: We can ask the professor. But he doesn't want to tell us exactly how to do it so it's hard to know whether or not we have the right answer at the end.*

While students were overwhelmingly positive about working in groups, they still experienced frustration. Causes of frustration most often mentioned were the professor’s style of answering questions (see section 4.4), a lack of examples (see to section 4.2), and the fact that many students felt, even after working through problems and arriving at answers, that they did not know if their answers were right until after their homework had been handed in and graded. The following student talked about this:

> *I: In terms of the way the course is being conducted, is it very different from your other classes?*
>
> *R: Yeah. It’s really hard to compare to anything because in a lot of my other classes we have lectures and we write papers. But here -- we get the materials, we work the problems, and then we find out if we’re right or not -- after we go through all of the frustration, then we find out if we’re right or not rather than just having everything laid out for us and having to summarize it.*

Interestingly, students did not mention other possible resources, such as text books.

Evaluators’ Viewpoint:
We observe that dependence on themselves and their peers as resources is an effective learning strategy for students. The faculty might want to decide whether it is a goal to have students recognize and use other resources as well.

4.6. Evaluators' Summary

- **What's working well**

For the most part, the structured group work is facilitating a successful learning experience. It is our opinion that several factors contribute to this outcome. The problems are challenging, setting up a situation in which no single student feels capable of solving the problems on his/her own. Thus the students have group interdependence - a critical component in cooperative learning.[3] That is, the students depend on each other as resources for solving the problems. Another factor is that the students must enter a dialogue regarding the problems. This interchange in and of itself is beneficial, because through the act of clarifying points to others, one develops a clearer understanding for oneself.

The following two points that we make about the value of cooperative work are not explicitly supported by the transcripts. That is not to say that the transcript data in any way contradicts them. But, we did not ask about, and students did not focus on the kinds of detail which would have addressed, these issues. (1) We assume that through dialogue, students use many mediums to make their points. Processing information in various ways -- aurally, visually (looking at diagrams), kinesthetically (drawing diagrams, gesturing -- functions to increase access to memory and conceptual understanding of the problems. (2) It is quite probable that a certain amount of anxiety and frustration normally experienced by these students when faced with challenging math problems is dissipated because the students are "in it together." Cooperative work functions to lower their "affective filters."[4] An affective filter consists of emotional and psychological barriers which inhibit the student from learning in an environment that is perceived as threatening and/or intimidating.

- **What could work better**

In the evaluators' opinion, students' perceptions that the professor doesn't answer questions indicate a weak area in the learning experience. It is interesting to note that while many broadly identified the professor's goal for his teaching strategy, and admitted that arriving at answers for themselves is of benefit, they nonetheless did not articulate the function of this teaching strategy in the learning process. For instance, a few identified the professor's responses as an attempt to lead them in the right direction. And many said he doesn't answer questions because he wants them to find the answers for themselves (these students did not identify the responses as "leading"). What the students don't identify is that the professor's responses contain information that they can use to strategically choose their next step in the problem-solving process. Students may experience deeper understanding if they learned to recognize the responses as information. This skill may

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[3] Johnson, D.W., Johnson, R.T., and Holubec, E.S. (1990). Cooperation in the Classroom (rev. ed.). Edina, MN: Interaction Book Company.

[4] English as a Second Language (ESL) is a widely studied field in education. For reasons associated with such factors as insecurity, low confidence, and fear, students learning a second language, or learning other subjects in a second language, have high affective filters. Various teaching strategies, such as cooperative learning, address ways to lower a student's affective filter. (Krashen, S. and Terril, T. (1993). The Natural Approach: Language Acquisition in the Classroom. Hayward, CA: The Alemany Press.)
A relevant paper, also dealing with this phenomenon, is: Steele, Claude M..A Burden of Suspicion: How Stereotypes Shape the Intellectual Identities and Performance of Women and African-Americans, Stanford University

even lead some to ask more strategic questions. (And as well, any students using these skills in group discussion will help others begin to use them.) While we feel these skills would be beneficial to the students' learning processes, we also feel that the nature of the professor's responses to students' questions (that is, the responses are information) may need to be pointed out and the skills for using the information may have to be explicitly taught.

The process of problem-solving we are talking about is the same process used by a player in the game *Master Mind*.[5] A good player will use the information generated from each guess to make better and better guesses. In fact, the player can use strategic guesses to get specific information, which allows for more informed guesses at each turn. At some point, the player is not actually guessing, but using logic and strategy to solve a problem. However, players can play the game by purely guessing at each turn. (In this case, the information generated by each guess would be interpreted as only "wrong" or "right".) It may take time and/or explicit training for these players to learn the needed skills to make logic-based and strategic guesses. We feel a similar problem-solving process can be used in the cooperative Math 141 groups. That is, they can learn to see the professor's responses as information which can inform their next steps.

5. OUTCOMES

Many students reported that they are experiencing math in ways they hadn't before.

- **Problems facilitate learning**

> *[Math 141 has changed how I think about math.] Because, these problems are structured [in a way that makes] you learn.*

- **Context and application**

[5] *Master Mind* is a board game. One player chooses three pegs (each of different colors) and orders them (by putting them into holes in the game board). The other player goes through a series of guesses, trying to get the right colors and the right location of each color, in the smallest number of guesses. After each guess, the first player responds with the number of "hits" (a correct color in the wrong location) and "bull's-eyes" (a correct color in the correct location). A version of the game can be played by guessing three-digit numbers. The following illustrates the use of logic and strategic guesses based on information gained at each turn:

Player 1	Player 2's guess	Player 1's responses to player 2
529	408	a) 0 bull's-eyes, 0 hits (4,0, and 8 are incorrect numbers)
	391	b) 0 bull's-eyes, 1 hit
	596	c) 1 bull's-eye, 1 hit (5 or 6 must be a bull's-eye) *It can be reasoned that the only possible choices remaining are 156, 563, 5_9, or 9_6, where "_" represents a number which has not yet been used.
	156	d) 0 bull's-eyes, 1 hit (This means 5 was the bull's-eye in guess c. Which means it must be the hit in this guess. Which means that 9 was the hit in guess c. That leaves 5_9. All that is left is to go through the numbers which have not yet been used.)
	529	e) 3 bull's-eyes

You can apply it to everyday life and things like that. And I guess I have a better understanding of some of the other courses I've been taking -- you know, gosh! I wish I would have had this earlier because I would have had a little easier time figuring out the probability in genetics or something like that. Or I'm taking a chemistry class now and there're a few things that I can put into perspective a little easier because I've now learned to pull apart a text problem or a numerical problem.

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I would say that I look at math differently now. I see more practical applications of [math] than I ever did before. I see math as more worthwhile. There are more applications for math than I thought there were.

****

I can see now how problems that we've done, problems that we've looked at, could actually come up. That there could be problems that I never thought would surface that could be solved with math. (…) I used to think that balancing your checkbook and figuring out if you were owed more money, about a credit card, or interest earned at the bank was the only way you would really use math. But I can see how some of the things we've looked at so far could show in practical applications of math that I didn't think of before.

Included in their new way of dealing with math is more logical and analytical problem-solving. The following student talked about this, and stated that s/he now sees more approaches to solving a problem:

I: OK. So what do you think you're learning in this class?

R: I think more logically about things. For instance, there was one section where he said: "How many people do you think are in the building right now?" And I thought, "How am I supposed to know that?" And now after we've gone through it, I know how to do that. And I guess, things like that, I'm really happy that this course [deals with] things that I wouldn't normally think about. (…) I'm learning that-- a problem can be solved different ways. Yes there might be a step by step way of figuring it out, but there are also different ways of estimating. There are different ways of figuring out problems, and I guess I didn't, I didn't really think about math in that way, before this class.

The outcomes of this section of Math 141 were overwhelmingly positive. In spite of the successes students accomplished, and the confidence they gained in their ability to approach challenging problems, most also experienced frustration. The following student described the difficulty of the course and the frustration s/he experienced, concluding that it was all for the best:

R: And you just have to figure out [the parts of problems the professor doesn't divulge].

I: Do you like that approach to trying to learn math?

R: It's difficult but it will probably help in the long run with problem-solving.

I: Do you feel like it has helped in the short run? What's your perception of what you're getting out of this class?

R: Of course I'd like somebody to tell me, "Here's the equation, now go solve the problem." But I mean they're not, and, that's just fine. But to have to work towards it, [finding the equation on your own], is a lot harder, and now I know how to do it and, yeah, it helps me.

While most of the students interviewed said they liked Math 141, benefited from it in some way, or at least preferred it over the alternatives, at least one student did not think s/he profited much from taking the course. When asked if her/his problem-solving ability had changed any, this

student replied that it had not because “if I can’t figure [something] out I just keep on trying different ways and that’s what I’ve always done.” However, even this student noted that working to solve problems as a part of a group was a significant and beneficial change in his/her approach to problem solving.

6. CONCLUSIONS

Most of the students interviewed from this section of Math 141 described significant successes related to their investment in the course. As one student said: “I find this math class more interesting than other math classes I've taken. (…) I think the skills taught by this teacher are valuable.” Many of the students described improvements in their attitudes toward math and a realization that math could be put to use in their other courses and in their lives in ways they had not previously recognized. A number of students gained greater confidence in their ability to approach and solve mathematical problems. One of the most striking findings was the important role that group work, combined with challenging home work problems, played in the learning process for most of these students.

Many students, even those who said they liked the course or recited benefits they derived from it, described varying levels of frustration with the way the course was taught. For many, not having direct answers to their questions and/or not knowing what procedures to use for solving problems was especially irksome. What is noteworthy about this is that many students, perhaps somewhat grudgingly, admitted that they recognized why the professor employed such a strategy: to force them to think things through and to figure it out on their own or as a part of their groups. Other students, perhaps less able to make this connection about *how* they benefited, at least stated that they were somehow able to learn through this process even though they could not necessarily recount how it was working. The following dialogue is representative of the challenge faced and the frustration felt by many students:

I: Did [Math 141] meet your expectations of being math that you can apply to the real world?

R: Some of it has, but it's difficult because I'm so used to a math class being like: "OK, here's the formula, this is what you do," and having a book that I can follow examples. And I don't like that about this class, because I feel there're no examples that I can go off of.

I: There's no process that you can learn how to use?

R: Right, right. Because that's the way I learn, you know. I need just to say, "OK, here's step one, step two, step three, here's the answer." And it's not like that, so--

I: So what is it like?

R: {Pause} It's so strange, it's like, you come up with it any way you can. I mean, make your own formula -- Just see if you can create [a formula], and in some way figure out this problem.

I: And you're not enjoying the challenge of this?

R: No, well, I enjoy it, but it's just, it's hard for me to think like that.

I: OK,

R: I guess is what I'm trying to say, it's difficult because I'm so used to other math classes.

The fact that students did make advances in their approaches to problem-solving and in making application to other things must be considered a significant success. The comments of so many of the students that group work was successful in breaching the void left by the professor's strategy to purposefully *not* give them direct answers to their questions is equally noteworthy.

One speculation which we would submit is that some of these students were unable to recognize that the professor set up the learning process and utilized examples as opportunities to teach the students a "model for problem-solving" as opposed to exercises in solving specific problems. This was evidenced in comments from students who did not see relationships between examples and other problems, who questioned why so few examples (of the same type) were given, and who did not see the value of solving problems that were not identical to ones they later saw on quizzes and exams.

An additional conclusion that we make is that no matter how beneficial the course, the students who enroll are unlikely to dramatically change their overall attitudes about mathematics. Nor are they likely to change their established behavior patterns of avoidance of formalized course work in the subject even with a relatively positive experience through their participation in the course. There was an odd juxtaposition in the attitudes of many of these students: they liked the class, many were glad they took it, and some said they would take it again if given the choice. But most of these same students claim they still do not like math and will try to avoid it in the future.

In conclusion, then, this section of Math 141 was, for most of these students, a significant success in getting them to look at math and its applications in a new way. While students expressed frustration because the course deviated from what they were used to, the routine use of groups provided a supportive environment in which students rose to the challenge of solving difficult problems. Indeed, the lesson that there were different ways to solve problems was an insightful one for some of the students. The most common source of frustration for students was the

professor's insistence on not answering their questions directly. Many students indicated that they understood this strategy. Those few who did not might have better understood the usefulness of this strategy had it been clearly and thoroughly explained to them. As it was, their lack of understanding about the process seemed to cloud their abilities to fully profit from it. As indicted above, however, even many of these students derived some growth in skills or attitude toward use of the subject.