Math 141
Final Report:
A synthesis of the findings in four Math 141 pilot sections offered during the 1996-1997 academic year

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Table of Contents

1. INTRODUCTION

Four different sections of Topics in College Mathematics (Math 141) were piloted during the 1996-1997 academic year. Each section was taught by a different professor, and the four sections were not coordinated. Math 141 is designed as an alternative to Math 112 (Algebra) in fulfillment of the first component of the university's general education requirement in quantitative reasoning (QR-A). Generally speaking, Math 141 is aimed at students who do not plan mathematics or mathematics-intensive majors; its general goal is that students will engage successfully in quantitative reasoning and will be able to use elementary mathematics in order to solve real-world problems.

The LEAD (Learning through Evaluation, Adaptation and Dissemination) Center on the University of Wisconsin-Madison campus produced formative evaluation reports for each section. These reports were based on classroom observations and, in particular, on the inductive analysis of exploratory, semi-structured interviews with students.[1] This summary is based on the findings in the four Math 141 reports. We discuss the types of students most likely to enroll in Math 141, and the challenges they face as math students. Then we discuss the various strategies employed by the Math 141 faculty and the resulting outcomes.

2. CHARACTERISTICS OF MATH 141 STUDENTS

The majority of students who take Math 141 are those who are seeking an alternative to Math 112 (or to Math 101 (Intermediate Algebra) and Math 112) for satisfying the QR-A requirement. These students identified themselves as not liking math (specifically, not liking the previous math experiences they've had), not being interested in it, and not seeing themselves as particularly good at it. However, all students *were* able to develop skills and strategies that helped them do adequately or better in their high school math courses. In fact, a small number of students felt they had strong math skills not adequately reflected in the placement tests which put them in Math 101 or Math 112. The primary reasons students gave for taking Math 141 were to satisfy the QR-A requirement, and do so while avoiding college algebra. It should be noted that some of the students wishing to avoid another algebra class also mentioned they were interested in the different approach offered by Math 141.

It is of note that some students who liked and were interested in math were attracted to Math 141. These students indicated that they liked the idea of getting a broader, more conceptual approach to math. The Spring semester offerings of Math 141 attracted a number of these students.

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[1] LEAD researchers attempted to invite all students to participate in the interviews (and most were reached). All who agreed were interviewed. The number of students interviewed from each of the four sections is as follows:

Class size	Number interviewed	Percentage of class total
22	9	41%
34	14	41%
36	12	33%
37	12	32%

A significant factor to be considered is the wide range of math skills and comfort levels which students bring to Math 141.[2] This presents a challenge to the faculty.

3. HURDLES FACED BY MOST MATH 141 STUDENTS

Most students who take Math 141 come to it with problematic intellectual habits and affective barriers. Most said they don't like math. They don't want to take Math 141 (or any math class). Math classes engender feelings of frustration. As well, Math 141 students do not necessarily have strong basic math skills needed to solve word problems. In fact, many describe themselves as weak math students. These students, weak skills or not, managed to develop intellectual habits that got them through their more traditional high school math courses. They've learned model solutions for solving various problem types. In general, learning strategies are structured around the use of a math text: chapters and problem sets organize various problems, which in turn help students to identify the model solutions to be applied. However, the strategies they relied on to get through past math classes are not necessarily helpful in Math 141. Yet, these strategies can be difficult to let go of when faced with unfamiliar learning strategies expected in Math 141.

4. WHAT STUDENTS NEED TO MEET MATH 141 GOALS

The broad goals of Math 141 are that students will engage successfully in quantitative reasoning and become independent and more confident problem solvers. Supporting goals identified by faculty include some of the following: students will critically discuss the mathematical content of articles and essays; students will correctly interpret statistical information and information presented in graph form; presented with a word problem, students will identify pertinent information, relevant equations for solving the problem, and any assumptions which will be made in order to solve the problem; and students will engage in a problem-solving process through dialogue with their peers.

4.1. Successful Strategies

As suggested above, most Math 141 students come to this course with intellectual habits and affective states that pose major challenges for the faculty who seek to help them become independent mathematical problem-solvers. Two issues stand out as having the greatest potential for inhibiting these students' Math 141 learning experience: the strategies which have worked for them in high school (rote application of algorithms and reliance on model solutions); and the apprehension and frustration they experience with math in general, and in particular with the new and unfamiliar challenges of Math 141. The strategies used in the four Math 141 courses offered during the 1996-1997 academic year were successful to the degree that they dealt effectively with these two issues. Upon

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[2] This was partially addressed by assigning one section, each semester, a Math 112 ready requirement. However, a small number of students who were not Math 112 ready were allowed to enroll in these sections. As well, some Math 112 ready students enrolled in sections which were open to those who had tested lower on the placement exam.

analyzing these successful strategies, we found that all of them put in place certain structures. These structures act to either provide impetus for engagement in certain types of learning, or define the boundaries of the learning experience. Below, we describe these engagement and boundary strategies in terms of the structures they put in place.

4.1.1. Challenging Problems

Some Math 141 courses used intentionally challenging problems to engage students in independent mathematical problem-solving. (One student, referring to the challenging homework projects, stated that one of those problems was equivalent to 15 textbook problems.) Students made it clear that they wanted some traditional support structures to aid them in dealing with these problems. Some of the requests included: equations presented up-front; problem sets of similar types (same model solution, different numbers); and, if not a text book, at least some text-like organization of the material (problems organized by category, examples, and problem sets). Despite requests for traditional support structures, the challenging problems used in Math 141 provided impetus. They helped to engage students in ways that enabled them to rely less on their familiar strategies.

An important component of the pedagogy which helped to make this engagement strategy work was that some professors also defined boundaries. They routinely refused to set students up with the traditional, more familiar learning structures they asked for. (However, it should be noted that some used a "scaffolding" approach in their pedagogy; that is, students were assisted – usually upon their request – in using their familiar approaches to problem solving. This is illustrated in section 4.1.3.) Students *had* to tease pertinent information out of the problems. They *had* to explore the possible relationships between variables, attempting to discover equations which would help solve the problem. And, because students were faced with a variety of problems, all different in some way, they had the impetus to learn how to generalize the problem-solving process in which they were engaged.

Three of the Math 141 sections used a lecture format interspersed with question-answer sessions. As well, some of these sections occasionally used formal groups to work on problems. During the question-answer parts of the lectures, and during formal group work, professors invoked the boundaries which forced students to engage in the type of learning the faculty intended. For example, students' tendencies to ask technical questions (What equation should I use?, What variables do these numbers stand for?) would induce responses which reminded them that they were to be engaged in the problem-solving process. As such, the professor would provide guidance by use of hints and suggestions. Students might be asked to think about the relationship between two variables (i.e. between distance and time traveled), or to draw a diagram or graph. Many students reported that they were explicitly taught to slow down, and not to "jump right in" but to think about the problem. The boundaries set up by this kind of guidance kept students engaged in a problem-solving mode, and kept them from reverting to looking for model solutions.

4.1.2. Combining Challenging Problems and Group Work

An interesting outcome regarding use of challenging problems is that many students (more in some sections than in others) learned to work cooperatively with peers in order to complete challenging homework assignments. Aspects of group work students cited as beneficial included: generation of more ideas on how to solve a problem; alternative explanations and explanations at a level they more easily understand; and that there is usually someone who can move the group forward when others are stuck. These aspects of group work functioned to support students through the learning process, while the resulting synergy acted to intellectually engage students.

The use of group work was pronounced in two of the sections. In one section, students met informally outside of class. In the other, group work was part of the formal structure.

Whenever group work is used as the mode for learning, the question of whether all the students are learning arises. The educational research on cooperative learning reports that certain conditions determine the likelihood that all students will be involved in the learning process. One of these conditions is positive group-interdependence.[3] This condition is set up when students in a group depend on each other for their own success. Based on the Math 141 interview data, the challenging homework problems facilitated group-interdependence.[4] No single student had all the knowledge or skills to work independently on these problems. They depended on each other. This was very clearly illustrated by the students who developed study groups outside of class. Although the professor encouraged students to work in groups, it still required initiative and coordination on the students' parts to make this happen.

One section of Math 141 used formal groups in class. This structure was such that, after a short lecture introducing new material, student groups spent the rest of the period solving challenging problems. The main difference between this learning structure, and the informal one described above, is that this one incorporates a "guide on the side." That is, the professor is available to answer questions, and to give hints and suggestions. With this built in resource, students have an option in the event that they are stuck or otherwise find themselves chasing solutions down dead ends.

In both sections, it should be noted, students spoke very highly about the benefits of group work. Many students credited the group as being critical to their learning experiences.

An interesting result from the formal group work section was that students were ambivalent about the professor's responses to their questions. (It would be

[3] Johnson, D.W., Johnson, R.T., and Holubec, E.S. (1990). Cooperation in the Classroom (rev. ed.). Edina, MN: Interaction Book Company.

[4]These same findings appear in the LEAD study of the WES Program, the Math 130-132 sequence (for pre-service teachers), Chemistry 110, and Introduction to Engineering (EPD 160).

misrepresenting the data not to note that students in all four sections shared this ambivalence to at least some degree. But it was particularly noted in this section, most likely due to the nature of the course structure.) Many students reported that the professor never answered their questions. By this they meant that he never said, "Yes, you are right", or, "No, you are wrong." Students' descriptions of the professor's responses ranged from "throwing us off track" to "getting us to see the problem from different perspectives," and from "sending us back to square one" to "leading us to the answer." The professor created a boundary with respect to the type of information students could elicit. By enforcing this boundary, the professor kept students from reverting to familiar problem-solving approaches and encouraged cognitive engagement and exploration of various ways in which they might solve the problem. Considering the success of this particular section, it is our interpretation that the professor's responses did provide some guidance for the student work groups. Students' ambivalent interpretations of the professor's responses most likely reflect a tendency to fall back on old intellectual habits – rote methods of problem-solving – when they felt frustrated. (We conclude that these students did not know how to interpret or best use the professor's responses. It is likely that these skills need to be explicitly taught.)

4.1.3. Graduated Transition

Some of the sections used a structure which facilitated a graduated transition from students' familiar approaches to math to a more conceptual problem-solving approach. We refer to this as scaffolding. By providing just enough information, allowing students to approach problems with some confidence and less frustration, scaffolding acts to engage students in problem-solving. Some sections incorporated more scaffolding than others. Course components which provided scaffolding fell into two categories: scaffolding provided by the course material and scaffolding provided through lecture or question-answer. Other components which provided impetus in these sections were real-life situations presented in context, to be solved or discussed. When using scaffolding to help students transition to new skills, the boundaries are not as rigidly observed, as was discussed in the sections dealing with challenging problems. In fact, in this case boundaries take on a different nature. Rather than keeping students from using familiar strategies (a boundary in the strict sense), they support students in their problem-solving efforts as students strive to learn and apply unfamiliar strategies (like a buoy).

- **Course material**

Some scaffolding was provided by a traditional text or by materials organized in the form of a traditional text. That is, problems were organized by chapter or problem-type. Problems organized in this manner allow students to identify the type of problem, and possible types of solutions based on the chapter or section. Material organized in this way also provides examples and problem sets which allow for practice of problem types. Two sections used traditional texts, and at least one section supplemented materials with packets containing explanations of problem-types with examples.

- **Lecture**

Another form of scaffolding is embedded in the professor's pedagogy. Scaffolding is apparent in the degree to which the professor is willing to provide algorithmic instruction on how to solve equations. (This degree of scaffolding may be considered remediation. Many mentioned that students requesting this kind of scaffolding were expected to see the professor during office hours.) Scaffolding is also apparent in the degree to which the professor will help set up the problem-solving process – identify the problem type and identify variables and relevant equations.

The analysis of interview data suggests that the four professors varied in the degree of scaffolding they offered in their pedagogical style.

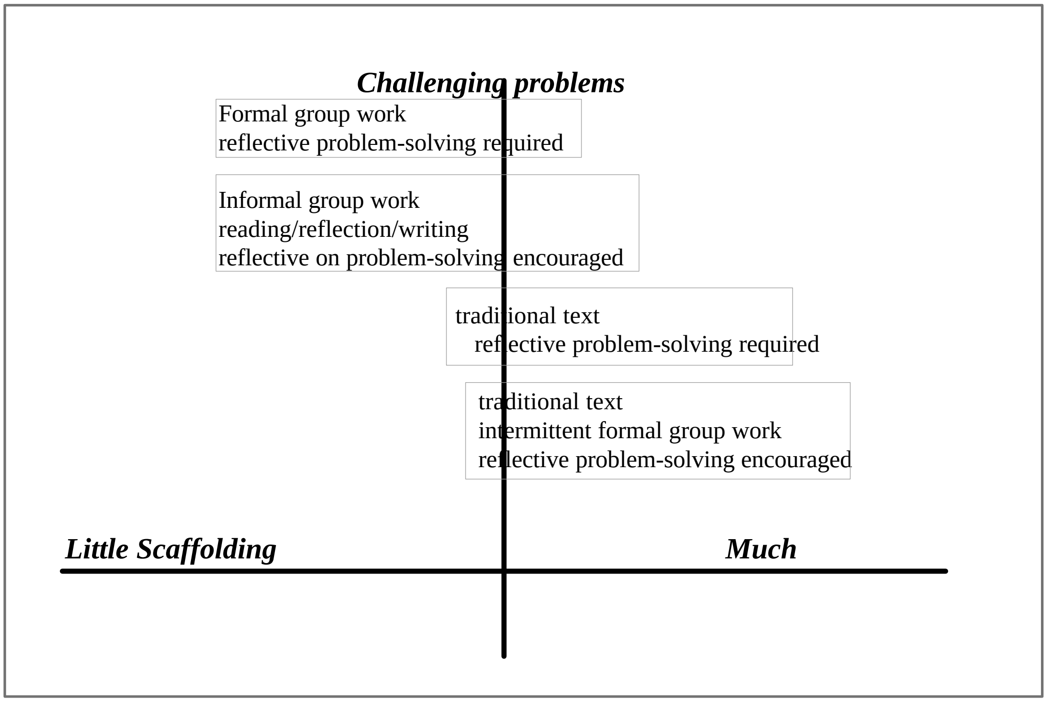
4.1.4. Activities Involving Cognitive Engagement

While one of the goals of the course is that students will engage successfully in quantitative reasoning, the four sections employed several strategies which, in the students' views, encouraged "mathematical thinking" (as well as cognitive engagement in problem-solving.) Outcomes and experiences students described included: learning to think more logically and analytically; learning to "take apart" a word problem and identify the pertinent information; and learning to interpret statistical information and information in graphs. While this last outcome possibly reflects the quantitative reasoning goal, student *reports* focused more specifically on correct application of mathematical processes (which we, in general, refer to as "mathematical thinking"). We interpret that the goal of quantitative reasoning skills, or "numeracy", is new to most students who enter Math 141 with twelve years of math education in tow. Not only do students fall back on familiar strategies (as mentioned earlier), but they approach the learning experience with pre-formed expectations of what math *is*, and of what *one is supposed to learn* in a math class. Therefore we conclude that the goal of quantitative reasoning skills needs to be explicitly defined and repeatedly emphasized to the students.

The course strategies included both discussion and writing. In discussion, peer groups worked together to solve problems, or large groups discussed mathematical content of essays, or analyzed statistical information or information presented in graph form. Writing was used in a couple of ways. In all sections students were encouraged, on their homework, to explain their problem-solving process. In fact, this written component of the homework was required in at least two sections. Some students reported that they were also encouraged to explain their process on exams. One section incorporated writing (as well as reading and discussion) in a way which is unique to math courses. It assigned various readings for discussion. The large group explored the use of statistics and logic in these essays and embarked on a critical analysis of their uses. Students were encouraged to bring in articles for the purposes of similar kinds of discussion. Students were assigned to write essays, using analytical and quantitative reasoning to reflect on various assigned readings. (A very interesting outcome from this section should be noted. Many students reported that the assigned book was interesting and fun to read, but stated definitely that it was not a math text, and should not be used in a math class. Many students did not identify the kind of analysis they were engaged in as math.)

The diagram in figure 1 is an attempt to visually organize the four sections in terms of two components (challenging problems and scaffolding), and further delineated by other components (group work, reflective problem-solving, reading, writing, and use of text).

Figure 1. The four sections of Math 141 are represented on the Cartesian system below. The vertical-axis represents the degree to which challenging problems were used and the horizontal-axis represents the degree to which frustration was managed through providing scaffolding. Each section is represented by a rectangle, inside of which are listed some course components.



4.2. Areas that can be Improved

While the sections enjoyed some significant successes, there are areas where improvements can be made. Almost all the improvements can be understood under the framework of dealing with students' affective filters. With a high affective filter, students experience emotional and psychological barriers that impede them from engaging in learning. As well, it is likely that students become more insistent on returning to familiar pedagogies when their affective filter is high. We suggest several ways in which faculty can help lower the affective filter. We don't suggest that all strategies, or any particular strategy necessarily be used. We do suggest that some combination of strategies, intended to provide a support structure for Math 141 students, be used.

Some strategies we suggest were already employed in some sections. 1) Encourage group work, and even formalize it (keeping in mind that group-interdependence is a necessary condition for optimum cooperative work and student learning). 2) Engage in some degree of scaffolding.

Other suggestions are based on the fact that Math 141 students have a wide range of math skills and confidence. As a result, while some students may very well be able to meet the challenge of quantitative reasoning (or "mathematical thinking", as the challenge seemed to be understood by students) and problem-solving, they may not be able to follow a problem through to completion due to lack of technical skills. Interview data revealed that many students understood it was the analytical thinking process that they were to be learning, and therefore did not experience apprehension over technical errors they might have made. Other students did not seem to understand this. We suggest that the goals of Math 141 be well defined, and explicitly communicated to the students. Another suggestion is that faculty explicitly teach the problem-solving processes the students are expected to learn. Interview data revealed that students experienced confusion and frustration in a number of circumstances: some did not understand how to write a critical analysis and wanted to know the process for doing so; some did not know how to breakdown and analyze the information in a word problem; and some did not know how to best use professor's answers to their questions when the answers were in the form of hints and suggestions.

5. EVALUATORS' SUGGESTIONS

Based on the analysis of interviews from four sections of Math 141 we suggest the following:

- course goals be clearly identified and made explicit to the students, especially the over-arching goal of quantitative reasoning
- course components be chosen that will provide a suitable balance of engagement and boundary structures for the students
- math skills be explicitly taught: approaches to problem-solving, critical analysis, interpretation and use of information/resources

6. SUMMARY

Math 141 was challenging to most students, and many remained frustrated throughout. The body of this report highlights the specific areas where students faced the most challenge and frustration; and the report offers suggestions for improving students' experiences and outcomes.

Despite students' frustration, many also reported a number of positive experiences and outcomes. Through the employment of various engagement and boundary strategies, all four sections of Math 141 enjoyed a number of successes. Students described themselves as better, more confident problem-solvers, as better able to think logically and analytically, and as seeing and appreciating a variety of math applications they hadn't before. A very noteworthy outcome is that students reported liking the class, or at least appreciating having to take it. Many reported having a broader understanding of math (that is, it is more than just plugging in numbers). And some indicated that they had a new framework for understanding mathematical processes. They began to focus on deeper understanding of the problem situation rather than plugging numbers into an algorithm. These students reported that it was their process and the validity of their reasoning that counted the most in Math 141.