Executive Summary

This document summarizes the findings of the LEAD Center report titled, "Final Evaluation Report: Math 130, Arithmetical Problem Solving." This summary is primarily intended for those who have not read the entire report, and presents no new analysis.

1. Background

1.1 Evaluation Methods

The evaluators relied heavily on qualitative research methods, including classroom observation and open-ended interviews with faculty and students, and supplemented resulting qualitative information with limited data drawn from surveys. It is important to note that qualitative and quantitative research methods differ not only with respect to data collection but with respect to analysis. Individual and focus group interviews and classroom observations allow the researchers to "get inside of" the experiences of these diverse participants. Data collection methods are as open-ended and participant-responsive as feasible to ensure that the experiences of the study participants, not the researchers, are reported. Likewise, analysis processes are fundamentally inductive to ensure that the participants' experiences shape the findings. In practice, this means that the researchers make every effort to at least temporarily suspend the ideas that structured their interview protocols and classroom observations. Analysis begins by reading transcripts with an eye to what is most important to the participants. In contrast to survey methods, these methods do not yield precise, quantitative assessments of the proportion of participants holding pre-specified opinions. However, these methods provide extraordinarily rich information expressing the complexity of the lived experiences of the study participants.

1.2 The Development of Math 130

In spring 1993 Robin Pemantle, a professor in the Mathematics Department, taught Math 120, "Theory of Arithmetic." At that time Math 120 and Math 122, "Real Numbers and Informal Geometry," formed the four credits of mathematics required for certification of pre-service teachers in elementary education. Mathematics faculty members primarily taught the course sequence using a lecture format and having students do homework from a required textbook. Professor Pemantle led an effort to re-structure the course. He and four other professors in both the Mathematics Department and the mathematics education section of the Department of Curriculum and Instruction designed Math 120 so that the focus was on problem solving and mathematical writing. The reformulation of Math 120 did not apply to all spring 1994 sections: one section did not use the new format. In the reformulated course, the students worked primarily in groups on difficult problems and then explained in writing the process by

which they found the solution as well as their reasoning underlying their answer. Professor Pemantle and Jen Szydlik, a graduate student in mathematics, taught the re-designed course in the spring 1994 semester.

In the fall of 1994, Professor Pemantle and the other members of the Math 130 committee enacted a change in the name and structure of the mathematics courses required for certification. This change was due, in part, to an action by the Wisconsin State Board of Education, increasing the math credit requirements from eight to twelve for certification of pre-service teachers in elementary education. Because the math methods courses within the School of Education remained at four credits, it was decided that the remaining math credits students earned through the Mathematics Department should be increased from four to eight. The credit load of Math 120 was increased from two to three credits and the course was renamed Math 130, "Arithmetical Problem Solving." The reformulated Math 120 course served as a basis for the format of all Math 130 sections. The geometry course was also increased to three credits and renamed Math 131, "Geometric Inference and Reasoning." Although the geometry course was not changed at this time, it was redesigned in the spring of 1995 with a structure similar to Math 130. In addition, a third two-credit course, Math 132 "Mathematical Models" was created as a requirement for elementary education certification. This course is slated to begin in fall 1995.

In addition to changing the structure of the Math 120 course, a format for preparing instructors to teach this new method was designed and implemented in fall 1994. The semester before teaching the fall 1994 course, prospective instructors regularly observed the instructors currently teaching the re-formulated Math 130. Several times over the course of the semester, the prospective instructors met as a group with the experienced instructors to discuss how they taught the course. In addition, while teaching Math 130 the instructors participated in an informal teaching seminar, which consisted of observing each others' classes and then meeting to share observations and experiences.

During the spring of 1995, Professor Pemantle was on leave and Ms. Szydlik piloted the new Math 131 rather than the Math 130 course. One instructor who taught the course during the fall 1994 semester continued to teach the course in the spring 1995 semester. As a result, two new instructors were asked to teach Math 130. The prospective instructors for the spring 1995 semester observed the instructors teaching Math 130 during the fall 1994 semester. The instructors communicated with each other informally throughout the semester and two of the instructors met once during the semester to discuss their experiences in teaching the course.

1.3 The Format of Math 130

Math 130 utilized a student-centered, inquiry-based problem solving approach. Although some instructors introduced new material using examples and definitions, there was no lecture component to the course. Throughout the majority of the class period, students worked in groups of three or four on problems out of a workbook created by Szydlik and Pemantle. The problems were fairly difficult and often had multiple solutions. When the instructor perceived that most of

the students had arrived at, or had come sufficiently close to, a solution, they formed a large group with the entire class and encouraged the students to discuss their findings. The class then examined the possibly different methods that individuals had employed in finding a solution. These large group discussions generally lasted until the end of the class period or until the students felt satisfied with the solutions and with any related questions that had emerged during the discussion.

As homework, students described in writing their methods of solving the problems worked on in class. The write-up included an explanation of the problem, the process by which the students tried to solve the problem, and a complete solution with an explanation of the reasoning underlying the problem. Most of the write-ups were done individually, although some were assigned to be done in groups.

All of the students took a midterm and a final examination. In the fall 1994 sections, the midterms were evening exams and were designed so that the students would not be pressured by the time constraints of a fifty minute class period. In the spring 1995 sections, the midterms were taken in class due to the fact that a time outside class was not feasible for all students.

2. Conclusions

2.1 Research Question 1

The primary question addressed in this evaluation is:

> *Are the Math 130 students learning a problem solving approach as well as acquiring an understanding of the mathematical content of the course? And if so, does this result in increased confidence in mathematical ability for the students of Math 130?*

Implicit here is a comparative question, "Are there differences in the nature of the student learning between the 'old method' and the 'new method' of teaching this course?" and, if there are differences, "To what factors in the new course can we attribute these differences?" Due to the small sample of "old method" alumni interviewed, we can not draw explicit conclusions about this comparative question.

Overview

Our evaluation findings suggest that the new format of Math 130 increased student problem solving skills as well as their understanding of the mathematical concepts covered in the course. Through interactions with group members and the open-ended nature of the problems that were assigned, students developed problem solving skills. Group work, in conjunction with writing about mathematics, played a significant role in ensuring that students understood the mathematical concepts underlying the problems and also that they acquired the skills to explain mathematics at a basic level. We note that the students themselves considered these skills to be directly applicable

to their future role as teachers. Many were excited about learning these skills and viewed this course as a model of a student-centered and inquiry-based approach to learning. In fact, some alumni of the new method that were teaching used these methods in their classes.

The effect of group work on student learning

Working in groups played a significant role in the student learning process in Math 130. In fact, almost every student interviewed preferred working in groups to the traditional lecture format. When groups worked collaboratively, students developed social relationships that provided a basis of support in which they felt comfortable discussing their ideas. Moreover, the group work played a significant factor in student learning. Students expressed that group work helped to increase their understanding of the material in Math 130 because they were:

- More engaged in the classroom setting and involved with the mathematics;
- Learning the material at their own level through explanations by their peers;
- Learning that there were multiple ways to solve, and multiple solutions to, problems; and
- Working through the problems without being given the method from the instructor.

In addition, through these outcomes of the group work, many students felt an increased confidence in their ability to do mathematics. However, we note that the supportive relationships that fostered a collaborative environment in the groups developed over time. Thus, some students who did not feel confident in mathematics found it difficult to switch groups because they needed to re-develop these social relationships within new groups. In addition, most students felt that switching groups too frequently inhibited the development of supportive relationships within a group and thus made it difficult for students to work collaboratively.

Interview data suggests that working collaboratively is a learned process and students need guidance as they adapt to a group-based learning format. When confronted with the different learning styles and mathematical background levels of group members, students tended to work individually or align themselves with those of similar ability. Thus, one part of student adaptation to working in groups was learning to work within a framework in which there are individuals with multiple levels of mathematical experience and varying degrees of confidence. In addition, many students felt that the instructor needed to play a role in facilitating their adaptation into working collaboratively.

The effect of mathematical writing on student learning

Many students expressed that the write-ups were an integral part of the learning process in Math 130 and complemented the problem solving in groups. Through the mathematical writing, the students sifted through and came to understand the material learned in the group, in their own individual way. In writing up the solution, students were able to recognize areas they didn't fully

understand and then focus on developing those areas. In addition, mathematical writing assisted some students in making a shift from focusing exclusively on finding the answer to understanding the process of problem solving.

While most students felt that the individual write-ups were beneficial to their learning, many disliked the group write-ups, because:

- They were too difficult to write as a group, due to students' varying schedules;
- They did not allow for the individual component of learning process; and
- Students needed to depend on the other members in their group for their grade.

The workbook problems and their role in the learning process

Most of the students that we interviewed considered the workbook problems to be appropriate for their level of understanding of mathematics. Many students felt that the problems were an integral part of their learning in the course and that they facilitated an exploration and deeper understanding of the problem solving process. However, many students noted that the problems assigned toward the end of the course were more dependent upon prior knowledge, and this was a factor in the tension in the group work and class discussion between students who had the mathematical background to solve the problems and those who didn't.

Applications to teaching

As preservice teachers, the students in Math 130 viewed the method and the material learned through the lens of applicability to teaching. Many of the students we interviewed stressed that this course had provided them with usable skills that they would eventually apply to their own classrooms. Some of the students found that the method of teaching presented in Math 130 was effective for both learning mathematics and modelling a way of teaching that they could apply to their own classroom.

- Many realized that their students would be more motivated and would learn the material better if they discovered it themselves.
- By experiencing multiple approaches and solutions to problems, many developed a deeper understanding of mathematics and felt that this would be influential to their own approach to teaching.
- Many learned how to explain mathematics at a basic level through group work and the write-ups.

Students were particularly critical of aspects of the course when they could not understand the applicability of the material to their teaching. This criticism arose in discussions about the types of problems used in the course; some students felt that the problems in the course were too abstract for elementary students.

The shift in the source of knowledge

In this new teaching and learning format, students were unable to rely on the traditional sources of knowledge such as an instructor or text. Students thus turned to other sources. For many students, groups provided a resource in which they constructed their own knowledge through collaboration with their group members. Through this process of working independently of an external authority, the students came to understand and know the material more deeply and were more engaged in the discovery of mathematical knowledge. In addition, through working together the students gained confidence by realizing that they could come to a mathematical solution on their own.

The adaptation of students to this new course format

Certain difficulties arose as students adjusted to the Math 130 teaching and learning style. Many students noted that this new course format was significantly different from what they had encountered in previous mathematics courses. We came to understand that many of the complaints and problems that the students repeatedly discussed were related to difficulties in adapting to the format of the course. We found that some students adapted rather quickly whereas others never felt comfortable. For some students, not having the instructor provide direct and concrete assistance on how to solve the problems was a large source of frustration. Many of the students who experienced these frustrations mentioned they lacked the background in mathematics to solve the problems. These students were very resistant to the new role of their instructor, feeling that the instructor should provide them with the background information and, if necessary, the method by which to solve the problems. In addition, many students wanted the instructor to provide support in their adaptation as well as more of an indication that they were on the right track in solving the problems.

Interview data suggests that the learning process in Math 130 will not thrive if teachers give too much help or too little help. By giving too much help, the instructor reduced or even eliminated students' motivation and engagement in the learning process. By contrast, when instructors left the class hanging too much, without providing support and feedback about whether the students were on the right track, the students developed negative attitudes toward the course and their own abilities in mathematics. It appears, moreover, that the instructor will discourage students from accepting and understanding this new learning environment by providing too little help and support in adapting to the new method. This is particularly true if the instructor, intending to push students to learn on their own, ends up leaving them with no framework by which to assess themselves. The instructor's ability to ask questions that provoke thought, to guide the students instead of letting them work completely alone, and also to reassure the students that their efforts are worthwhile has a significant effect on students, as demonstrated by an increased confidence in their problem solving skills and a more successful class experience overall.

Student attitudes toward assessment

Students, when faced with a new course format in which they worked exclusively in groups with little or no direct instruction, expressed concern over how they would be assessed. For many students, grades were a strong motivating factor and they were anxious to learn how they would be assessed in order to succeed in this new format. Some students, particularly at the beginning of the course, expected that the assessment procedures would match the new classroom format. This concern was manifested in student comments about the exams: students felt at the beginning of the course that the midterm and final examinations should be group-based, since the majority of the class period was spent working in groups. Interviews indicated that most students realized over the course of the semester they were learning the material in their groups well enough to succeed in the individual exam.

In addition, students also expressed concern about the problem set write-ups. Since the students had little or no experience with explaining mathematics through writing, many were unsure what was expected of them in order to do well in these exercises. Many students commented that they were unsure what the instructor wanted, and stated that providing an exemplary write-up would have been helpful. In addition, many students expressed concern that the grade they received on the write-ups did not always reflect their knowledge and understanding of the problem. Many wanted to know how the instructor determined grades on exams and write-ups and suggested that the instructor provide specific examples of "A," "B," and "C" work.

2.2 Research Question 2

An additional research question addressed in spring 1995 was:

> *What is the effect of the transition to a new group of instructors teaching the re-structured course?*

This question grew out of an interest in understanding what occurs when the faculty member that formulated and originally implemented the changes in a course no longer participates in the teaching of that course.

Goals of instructors

In part, due to Robin Pemantle's efforts to convey the philosophy underlying Math 130, there was general agreement among the instructors about the goals of the course. The formation of goals of Math 130 was influenced by the fact that the students were future teachers. The instructors' goals for the course were as follows:

- Changing students' relationship with math from one of fear to confidence in their mathematical ability;

- Changing students' perception of mathematics as a series of disconnected formulas that must be memorized for an exam, to a system of ideas that exist within a framework of logic;
- Increasing students' problem solving skills to the point that they can work independently of an external resource;
- Increasing students' knowledge of mathematical concepts through the process of problem solving and mathematical writing; and
- Increasing students' ability to speak about mathematics through mathematical writing and group work.

It is important to note that while the instructors shared these goals, they differed as to which goals they considered most important and also in their strategies for achieving these goals.
This became apparent particularly when discussing the goal of increasing students' knowledge of mathematical facts. Some instructors felt that the students should learn a specific body of knowledge, and that the problem solving process should be channelled, if necessary, to ensure that the students learn this material. Other instructors did not emphasize that students should learn specific facts; rather, these instructors focused on developing in the students a problem solving approach to mathematics. These instructors commented that students would develop a deeper understanding of mathematics through the inquiry-based process of problem solving and as a result, would not only increase their knowledge of mathematical facts they discovered, but also their understanding of the underlying reasoning that supports these concepts.

Teaching strategies and roles

Shifting responsibility for learning

All instructors worked to shift responsibility for learning from themselves to the students in order to get students to become mathematical thinkers. The instructors did this by "stepping back" from the role of providing the method by which to obtain the solution and thus compelling the students to "step forward" and find their own solutions to the problems. Our observations of the course and interviews with the instructors revealed that not all instructors shifted responsibility to the students in the same way or to the same degree. This was indicated by the amount of assistance the instructors provided to the students as they worked on problems.

Some instructors gave little or no assistance in the problem solving process to the students, explaining that help given in the form of definitions and examples contradicted their theory on how students learn. These instructors commented that giving too much help would:

- Prevent students from taking responsibility for learning;
- Reduce their motivation for solving the problem; and
- Attenuate their engagement in solving the problems;

In contrast, other instructors guided the students by giving help in the form of definitions, examples, and framing the objective of the problem. These instructors expressed that giving help

in these forms ensured that the students would find the solutions, while still allowing the students to experience problem solving.

Providing support and motivating students to learn in this new method

As part of their role in the course, the instructors provided support for students as they adapted to this new teaching and learning environment. Many provided feedback as to whether the students were on the right track, and encouraged students when they were frustrated. Some instructors attempted to orient the students to the benefits of this new inquiry-based environment. This served to facilitate the students' acceptance of, and motivation for, learning in this new format. In addition, while some effort was made to facilitate students' adaptation to working collaboratively in groups, our observations and interviews indicate few instructors actively addressed this issue.

Bringing out ideas in the classroom

The instructors considered the class discussion to be a forum which allowed students to experience the process of problem solving by working together as a class through the different methods the students employed to solve the problems. Thus, one role of the instructor in the class discussion was to draw out the key ideas that students came up with in the groups, thereby guiding students toward a solution. By re-enacting the steps in the problem solving process, the instructors perceived themselves as creating a forum in which the students could participate in the process by which ideas are formed and refined into an end product.

Transferring the philosophy of Math 130

Many instructors commented that a formalized way of transferring the philosophy behind Math 130 needs to be enacted. These instructors were concerned that the philosophy was not being communicated to new instructors. Some instructors felt that in order to respond effectively to student queries about the format of the course, the instructor must have an understanding of, and belief in, the philosophy of Math 130. These instructors explained that instructors who are unaware of the goals of the course may not be prepared to deal with students' frustrations and complaints and, in reaction, might begin to step back into the traditional role of providing the methods and solutions to problems.

The instructors commented that a formalized "hand off" method must include a teaching guide that would present the philosophy of Math 130 as well as more detailed information on implementation issues such as comments on those things the instructors found successful and unsuccessful about certain problems. In addition, these instructors expressed that instructors new to the method should observe experienced instructors teaching the course and discuss with them their teaching strategies and views on the course. A few instructors explained that a continuous dialogue between current instructors should take place in order to prepare instructors for unexpected issues that might arise and also to keep them focused on the goals of the course.

Experiences of instructors new to the Math 130 method

In addition to facilitating the students' adaptation into this new learning environment, instructors new to the Math 130 method experienced their own transition into a new style of teaching. A few described this transition as "scary and risky" because, unlike the traditional lecture format in which the instructor leads the class, they could not prepare for, and predict, what would happen in class on a given day. One instructor commented that the transition into a non-lecturing teaching style was difficult because the instructor enjoyed lecturing, and felt that the role of the instructor was to organize and explain information to students.

Many instructors commented that the student-centered format and flexibility of the curriculum in Math 130 gave them the opportunity and time to recognize and react immediately to problems with student understanding of the material. However, a few instructors commented that a possible weakness in the curriculum was that the material did not build upon itself and as a result, students did not learn any "deep" mathematical concepts and also did not feel a sense of mastery of the material.