Final Evaluation Report

Math 130

Arithmetical Problem Solving

Department of Mathematics, College of Letters and Science
UW-Madison

1994-1995

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prepared
for

Professor Robin Pemantle
and the
Math 130 Instructional Team

by
The LEAD Center

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Preliminary Issues

A. The Development of Math 130

In spring 1993 Robin Pemantle, a professor in the Mathematics Department, taught Math 120, "Theory of Arithmetic." At that time Math 120 and Math 122, "Real Numbers and Informal Geometry," formed the four credits of mathematics required for certification of pre-service teachers in elementary education. Mathematics faculty members primarily taught the course sequence using a lecture format and having students do homework from a required textbook. Professor Pemantle led an effort to re-structure the course. He and four other professors in both the Mathematics Department and the mathematics education section of the Department of Curriculum and Instruction designed Math 120 so that the focus was on problem solving and mathematical writing. The reformulation of Math 120 did not apply to all spring 1994 sections: one section did not use the new format. In the reformulated course, the students worked primarily in groups on difficult problems and then explained in writing the process by which they found the solution as well as their reasoning underlying their answer. Professor Pemantle and Jen Szydlik, a graduate student in mathematics, taught the re-designed course in the spring 1994 semester.

In the fall of 1994, Professor Pemantle and the other members of the Math 130 committee enacted a change in the name and structure of the mathematics courses required for certification. This change was due, in part, to an action by the Wisconsin State Board of Education, increasing the math credit requirements from eight to twelve for certification of pre-service teachers in elementary education. Because the math methods courses within the School of Education remained at four credits, it was decided that the remaining math credits students earned through the Mathematics Department should be increased from four to eight. The credit load of Math 120 was increased from two to three credits and the course was renamed Math 130, "Arithmetical Problem Solving." The reformulated Math 120 course served as a basis for the format of all Math 130 sections. The geometry course was also increased to three credits and renamed Math 131, "Geometric Inference and Reasoning." Although the geometry course was not changed at this time, it was redesigned in the spring of 1995 with a structure similar to Math 130. In addition, a third two-credit course, Math 132 "Mathematical Models" was created as a requirement for elementary education certification. This course is slated to begin in fall 1995.

In addition to changing the structure of the Math 120 course, a format for preparing instructors to teach this new method was designed and implemented in fall 1994. The semester before teaching the fall 1994 course, prospective instructors regularly observed the instructors currently teaching the re-formulated Math 130. Several times over the course of the semester, the prospective instructors met as a group with the experienced instructors to discuss how they taught the course. In addition, while teaching Math 130 the instructors participated in an informal teaching seminar, which consisted of observing each others' classes and then meeting to share observations and experiences.

During the spring of 1995, Professor Pemantle was on leave and Ms. Szydlik piloted the new Math 131 rather than the Math 130 course. One instructor who taught the course during the fall 1994 semester continued to teach the course in the spring 1995 semester. As a result, two new instructors were asked to teach Math 130. The prospective instructors for the spring 1995 semester observed the instructors teaching Math 130 during the fall 1994 semester. The instructors communicated with each other informally over the course of the semester and two of the instructors met once during the semester to discuss their experiences in teaching the course.

B. The Math 130 Evaluation Project: Who, What, and How

1. Who: The Research Team

This report is the final product of an eleven-month (August 1994 - June 1995) evaluation project, though some preliminary discussions and classroom observations took place in spring 1994. Funding was provided through Professor Robin Pemantle's Presidential Faculty Fellow Award administered by the National Science Foundation. The evaluation team, which worked under Susan B. Millar as Director of the UW-Madison's Learning through Evaluation, Adaptation and Dissemination (LEAD) Center, consisted of Baine B. Alexander, the Associate Director of the LEAD Center, as project director, Heather A. Lewis (working from August 1994 through January 1995) and Sue Daffinrud (working from January 1995 through June 1995), both graduate students in the UW-Madison Department of Mathematics, as research specialists.

2. What: Formative Evaluation

Evaluation is "formative" when applied researchers give faculty reformers feedback while the program/activity is being planned, piloted, and scaled up, with the intention of improving the program/activity while it is under development. Because Math 130 is a continuing program with new faculty becoming involved in the course, this evaluation is formative.

3. How: Research Questions and Methods

Research Questions

The primary question addressed in this evaluation is, "Are the Math 130 students learning a problem solving approach as well as acquiring an understanding of the mathematical content of the course?" And if so, "Does this result in increased confidence in mathematical ability for the students of Math 130?" Implicit here is a comparative question, "Are there differences in the nature of student learning between the 'old method' and the 'new method' of teaching this course?" and, if there are differences, "To what factors in the new course can we attribute these differences?" Due to the small sample of "old method" alumni interviewed, we can not draw explicit conclusions about this comparative question.

An additional research question to be addressed in spring 1995 was, "What is the effect of the transition to a new group of instructors teaching the re-structured course?" This question grew out of an interest in understanding what occurs when the faculty member that formulated and originally implemented the changes in a course no longer participates in the teaching of that course.

Research Methods

We pursued these research questions by using various qualitative research methods: structured open-ended interviews, observations of the course, and a survey of the students in Math 130. We collected multiple types of data so that we could triangulate across a range of different data sources during the analysis stage. In addition, we used a diversely trained research team consisting of an anthropologist and two math graduate students. This enhanced the quality of the research because individuals trained in different disciplines bring together different perspectives on the same data.

The reader should note that qualitative and quantitative research methods differ not only with respect to data collection but with respect to analysis. Individual and focus group interviews and classroom observations allow the researchers to "get inside of" the experiences of these diverse participants. Data collection methods are as open-ended and subject-responsive as feasible to ensure that the experiences of the study participants, not the researchers, are reported. Likewise, analysis processes are fundamentally inductive to ensure that the participants' experiences shape the findings. In practice, this means that the researchers make every effort to at least temporarily suspend the ideas that structured their interview protocols and classroom observations. Analysis begins by reading transcripts with an eye to what is most important to the participants. The primary analytical categories that emerge as the researchers process the transcripts are apparent in the table of contents. In contrast to survey methods, these methods do not yield precise, quantitative assessments of the proportion of participants holding pre-specified opinions. However, these methods provide extraordinarily rich information expressing the complexity of the lived experiences of the study participants.

Open-ended Interviews

In order to determine how the students experienced the course, we interviewed current students in Math 130 and individuals who had taken Math 120 in either the old or new method. The structured open-ended interviews lasted approximately forty-five minutes to an hour. The interview protocols for fall semester appear in Appendix A and those for spring semester appear in Appendix B. Most interviews were recorded and transcribed; an average transcription was twenty single-spaced pages.

We interviewed sixteen current students in the fall 1994 semester, twenty current students in the spring 1995 semester, and twenty alumni of Math 120 in fall 1994.

Between five and eight students in each of the six sections were interviewed. Twenty-six students were interviewed twice: once within the first four weeks of the course and once within the last three weeks of the course. Interviewing the students at the beginning and end of the semester allowed us to observe if and how the students' experiences and attitudes

changed throughout the semester. In addition, it also gave us the opportunity to examine and trace patterns that emerged in the first set of interviews. Most of these interviews were done on an individual basis, while some of the students were interviewed in focus groups.

Between October and December of 1994 we interviewed twenty alumni who had taken math 120 between fall 1992 and spring 1994. Of these, thirteen had taken it under the old method and seven under the new method. Eight of the old method students and three of the new method students were either currently teaching or currently student teaching. The number of student teachers interviewed was limited by the fact that of the fifty-eight student teachers in elementary education in fall 1994, only twenty-one had taken Math 120 within the past two years (twelve in fall 1992; five in spring 1992; two in fall 1993; and two in spring 1994). Five of the old method students and six of the new method students had taken the course the previous semester. These recent alumni were drawn from a list of individuals who, during their Math 120 course, had stated that they would be willing to be interviewed during fall 1994.

In addition, we interviewed Robin Pemantle in spring 1994, and each of the six instructors during the semester in which they taught.

Classroom observations

In the spring of 1994, one old method section and one new method section were observed twice. In fall 1994, a researcher observed each of the three sections at least twice; in spring 1995, a researcher observed each section three times. A sample observation from each section is included in Appendix C.

Survey

An anonymous survey was distributed in each of the sections in December 1994. The purpose of the survey was to determine if the findings from the qualitative interviews were representative of the classes as a whole. The students were asked to complete the survey outside of class and return it to the researcher at the next class period. There were thirty-four responses from the fifty-five enrolled students. Although information compiled from the survey was used to confirm and trace patterns that emerged in the observations and interviews, no analysis of the survey is presented in this document.

C. Glossary of Special Terms

Certain terms are used with precise meanings throughout the report. To clarify those meanings, the terms are defined here:

alumnus/alumni:
Alumni are people who have already taken Math 120. They may or may not still be enrolled as students at the University of Wisconsin.

current students:
Current students are those students enrolled in Math 130 at the time of the interview.

formative evaluation:
Evaluation is "formative" when applied researchers give faculty reformers feedback while the program/activity is being planned, piloted, and scaled up, and with the intention of improving the program/activity while it is under development.

Math 120:
Math 120 is the two credit math course "Theory of Arithmetic." Through spring 1994, this was the first in a sequence of two math courses required for certification in elementary education (pre-kindergarten through grade three, grades one through six, or grades one through nine).

Math 130:
Math 130 is the three credit math course "Arithmetical Problem Solving." Beginning fall 1994, Math 130 was the first in a sequence of three math courses required for certification in elementary education (pre-kindergarten through grade three, grades one through six, or grades one through nine).

new method:
The new method refers to the approach to teaching Math 120 and Math 130 as developed by Professor Pemantle and the Math 130 committee. All sections of Math 130 use the new method. In addition, those sections of Math 120 that were taught by Professor Pemantle and Jen Szydlik, a graduate student in mathematics, used the new method.

old method:
The old method refers to any approach teaching Math 120 other than the specific approach utilized by Pemantle or Szydlik, a graduate student in the mathematics department. Any sections of Math 120 that were taught by instructors other than Pemantle or Szydlik used the old method. It is important to note that while there were some similarities among the old method sections as taught by different instructors, there was no single old method as such. In addition, some elements of the new method, such as the use of large group discussion, appeared in some sections of Math 120 using the old method.

recent alumni:

Recent alumni are those individuals who took Math 120 in Spring 1994. They may or may not still be enrolled as students at the University of Wisconsin.

D. Notes on the Use of Verbal Quantifiers

Specific verbal quantifiers are used to denote the relative size of a group of participants who presented particular perspectives or described particular experiences in interviews. It is important to note that due to the nature of qualitative interviews, the size of a group who discussed a particular type of experience does not indicate the size of the group who had this type of experience. Although the same interview protocol was used in each interview, respondents' answers often prompted discussion in a particular area that may not have emerged in other interviews.

The verbal quantifiers used in this report are:

"a few":
used when up to 30% of those interviewed presented the perspective under consideration

"many":
used when 30 to 70% of those interviewed presented the perspective under consideration

"most:"
used when 70 to 90% of those interviewed presented the perspective under consideration

"virtually all":
used when 90% or more of those interviewed presented the perspective under consideration

"a subset""
used to articulate more gradations within a group referred to previously by "a few," "many," or "virtually all." A subset includes at least two individuals

E. Notes on Quoted Material

Ellipses (...) in quoted material indicated deleted dialogue occurring within the reproduced material. Deletions are made so that the readers can appreciate speakers' views on a particular topic without having to sort through the divergent twists and turns of the raw dialogue. Explanatory words added to quotes appear inside brackets [ ]. The quoted material is presented as faithfully as possible to the speakers' intent. Interview dialogue is marked "I:" to indicate an interviewer's speech and is marked "R:" to indicate the speech of the respondent. In interview passages in which more than one respondent is quoted, "R1:," "R2:," etc. is used.

From an Observer's Point of View: The Format of Math 130

We evaluated all six sessions of Math 130 that were offered in fall 1994 and spring 1995. Three sections were taught in fall 1994 and three were taught in spring 1995. In five of these six sessions, students attended three 50-minute classes each week. In the remaining section, students attended two 75-minute classes each week. All sections utilized a group work approach to problem solving and emphasized mathematical writing.

A. Common Features Among All Sections

Students in all sections of Math 130 were required to buy a packet of problems created by Pemantle and Szydlik instead of a textbook. The course was organized into seven or eight units which focused on such conceptual areas as "Place value and Other Bases" and "Number Systems." The amount of time spent on each unit varied, but averaged about two weeks. Students worked in small groups of three or four on problems designed by the instructors for each specific unit. The instructors generally switched the student groups monthly, thus students worked in three or four groups over the course of the semester. In one section, the students remained in the same group throughout the semester.

A class period generally began with collection of homework and general announcements. In some sections, if the class was beginning a new unit, the instructor provided a brief introduction to the material. Then students gathered into their small groups and worked on an assigned problem. The problems were fairly difficult and usually there were several ways in which to solve them. New problems were assigned every one to two class periods. The character of the individual groups varied. Some of the groups were quite vocal, with a lot of discussion among the members, and others were quieter with the students working individually a lot of the time and only occasionally asking each other questions. When the instructor perceived that the groups had a sufficient amount of time to work on the problems, s/he formed a large group with the entire class and encouraged the students to discuss their findings. The class would talk about the different methods that individuals had used to arrive at a solution. In addition, they would discuss difficulties that had arisen in the problem solving process. The instructor would either write students' ideas on the board, or the students themselves would go to the board to describe partial or complete solutions. These large group discussions generally lasted until the end of the period or until everyone felt satisfied with the solutions and with any related questions that had emerged during the discussion.

As homework, the students described in writing their methods in solving the problems worked on in class. The write-up included an explanation of the problem, the process by which the students tried to solve the problem, and a complete solution with an explanation of why the solution made sense. Most of the write-ups were done individually, although some were assigned to be done in groups.

There was daily variation in the classroom routine. If the math problem assigned was particularly difficult, the students might be asked to think about the problem over several days and would spend more than one class period solving it. Sometimes a class finished a problem in the middle of the period, and then a new worksheet would be started and continued at the next class meeting.

All of the students took one midterm and a final. In the fall sections, the midterms were evening midterms and were designed so that the students would not be pressured by time constraints of a fifty minute class period. In the spring sections, the midterms were taken in class due to the fact that a time other than during the class was not feasible for all of the students.

B. Differences Among Individual Sections

Despite a similar overall format, there were several differences between the individual sections. Although the instructors used many of the same worksheets, they varied the order of the worksheets they assigned. Instructors also wrote their own exams and some provided review sheets for students in preparation for the exams. All of the classes used group work, but the composition of the small groups was determined in different ways. The instructors also utilized different systems for weighting various aspects of the course in the determination of the final grade.

Section 1 (Fall 1994)

This section had 22 students. This class was taught by an instructor who had taught Math 120 using the new method. Final grades were based as follows: 15% on attendance, 55% on written work, 15% on the midterm, and 15% on the final. This instructor assigned two of the groupings randomly and the third so that no two individuals in a group had been together previously. In this last case, the instructor made an exception by placing two students who worked well together and were at the same level in the same group, even though they had been in the same group before. This decision was made so that the students would derive confidence from each other and not rely on better students during the problem solving process. The students in this section were allowed to use their notebooks during exams.

Section 2 (Fall 1994)

This section also had 22 students and was taught by an instructor who had taught Math 120 but had not taught using the new method. Final grades were based in the following manner: 20% on attendance and participation, 50% on written work, 15% on the midterm, and 15% on the final. At the end of the semester, however, the instructor felt that, because exam scores varied much more than homework scores, the final grade should have been based more heavily on the exams. The students were placed in three groups over the semester. The instructor assigned students randomly to the first groups, and composed the last two groups in such a way to have a mixture of abilities in each group. The instructor felt that groups composed in this manner worked better.

Section 3 (Fall 1994)

This section had 11 students and was taught by an instructor who had previously taught the course using the new method. Final grades were based 15% on attendance, 45% on written work, 20% on the midterm, and 20% on the final. This instructor assigned reflections, or mathematical essays that focus on general issues such as "What is Math?" that were not directly related to solving particular problems out of the workbook. Reflections made up a larger portion of the written work in this class than any other section. This instructor assigned all four groupings of students randomly.

Section 4 (Spring 1995)

This section had 27 students and was taught by an instructor who had also taught Math 130 during the fall 1994 semester. Final grades were based 40% on written work, 25% for the midterm, 25% for the final, and 10% for attendance and class participation. This instructor assigned two reflection papers, "What is Math?" and the other asking students to describe subtraction to someone that knows only addition. The students remained in the same group throughout the semester because the instructor felt that the students were working well together.

Section 5 (Spring 1995)

This section had 27 students and was taught by an instructor who had taught Math 120 using the old method, and had not previously taught Math 130. No reflections were assigned. Final grades were based 65% on participation and homework, 15% on the midterm exam, and 20% on the final. This instructor assigned the first grouping randomly, and the last two in such a way that each person would have a former group member in their new group.

Section 6 (Spring 1995)

This section had 34 students and was taught by an instructor who had not taught Math 130. No reflections were assigned. Final grades were based 10% on attendance, 50% on write-ups, 40% on exams. There was a grader for this section up until the twelfth week of class, at which time the paid grading hours were exhausted. The instructor then graded the assignments and asked the students to write the remaining assignments as a group. In this case, the students were allowed to select their own groups. These self-selected groups lasted until the end of the semester. Prior to this last grouping, the instructor assigned the groups randomly every four weeks.

There were also informal differences among the six sections. In all but section three, large group discussions took place with students seated roughly in rows, whereas in the third section, students got into a semi-circle for class discussions. In addition, with the third section and the sixth section, the instructor generally did not write any solutions on the board but had the students do this themselves.

C. Differences From Math 120

Students taking Math 120 attended three fifty-minute classes or two seventy-five-minute classes each week. Prior to the development of the new version of Math 120, there was no consistent manner of teaching the course. Different instructors used very different ways of teaching the course, some of which included aspects of the new method. The students we interviewed described the following classroom situations:

One of the classes was entirely lecture without discussion of the homework.

Two of the classes were primarily taught with a lecture format, with the instructor teaching the material and going over homework problems.

One of the classes started out as a lecture class with the instructor going over homework. Towards the end of the semester it switched to a small group format. At this point, the students worked in pairs on problem solving.

One of the classes used a lot of large group discussion. The instructor began each class by introducing a problem similar to the one in the text. The students would then brainstorm ways of solving it and talk about the different methods. Sometimes individuals would put their answers on the board.

In addition, all of the classes used a required text and all of the instructors used some class time as a forum to answer students' questions.

From the Students' Point of View: The Learning Process

In this section we present characteristics of the learning process in Math 130 as described by the students we interviewed and through the surveys. Many of these are themes that the students themselves initiated and discussed. Our discussion will focus on the following features of the course: group work, the problem solving process, views on assessment, and applications to teaching. It should be noted that we used quotes from interviews with new method alumni where they expressed views that were also held by current students.

A. Learning through Group Work

Working in groups played a significant role in the student learning process in Math 130. In fact, virtually all students interviewed preferred working in groups to the traditional classroom format of having an instructor lecture to them. Many different facets of the group work experience were discussed by the students we interviewed. In the following section we will focus on the most salient themes involving group work that arose out of our discussions with students.

1. Over time, group work provides a source of support

Many of the students that we interviewed described how, through the group work, they developed social relationships with the other individuals in their class. These students expressed that the group fostered a supportive environment which they found useful when problem solving. The following three student excerpts are typical examples of how the groups provided this support.

> I like the group work. I think it's a good idea. I like the fact that I can lean over and say, "Do you understand?" And somebody could say, "Yeah, I understand. Remember when we were --- ? Then it's just like that except for you have to do this instead." I think that definitely provided a lot of support, and it made the classroom just a better environment. It wasn't the traditional, competitive -- like, "I have to get an A, so I have to watch everyone else."...I think it was a cooperative environment, everybody's just helping each other out....I've made a few friends in there, and that's actually kind of a rarity in my college classes.

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> [What I liked about the group work was that] if you didn't understand something, you didn't feel like a failure if you couldn't get any of the problems. Because everyone was willing to help one another. And maybe things weren't really good when you started something, so you felt good about contributing at least a little bit. Just having people there to help you and realizing that others are frustrated.

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> I was mad I had to even take it. I mean, Math 130 after Calc 222 [second semester calculus]? I was like, "Forget this." But it is really fun, and you really develop a bond with a lot of

> people in the class. It's fun to go, even though the work is sometimes going to be a pain and [the problems] take me a long time to do.

Students expressed that having a supportive relationship with their classmates was particularly important in gaining confidence both to ask questions about the problems and to contribute their own ideas to the problem solving process. Most students felt that this supportive relationship developed over time.

> [Understanding and trust] really helps with group dynamics, because some people are naturally extroverted and want to take a lead in a group. And some people are kind of shy actually. I think that when you work together long enough, you kind of remove the peaks and valleys and you kind of bring everybody to one level and it becomes more of a group interaction thing. I think that the first time that you work together, everybody's natural tendencies come out. I tend to be very extroverted, and jump right in to say whatever is on my mind. But, after I work with some people for a while, you kind of step back a little bit, and try to bring other people forward and it tends to level the group off, and you become more efficient.

The effect of switching groups on the development of a supportive group environment

In five of the six sections, the instructors switched the groups at least three times during the semester. We will discuss in this section student views on how switching groups affected their relationships within the groups. While many students felt comfortable with switching groups and actually preferred having the opportunity to work with different people, some students found it difficult to switch groups because they had to re-develop these social relationships with the new group.

> I have a real low threshold for uncomfort in math. It was really hard for me to go in initially in a group of four other people that I didn't know and feel that unsure of myself and think, "If only they knew how stupid I am." And then finally working things out in that group so that you felt comfortable and realized that this is a pretty safe place here. They aren't going to think I am stupid. And then every couple of weeks having to change into a new group and new dynamics and new personalities. And any group of three to four people you just have a whole different -- and some of the groups felt more comfortable than others. Having found one group of four other strangers that we eventually developed a rapport. I know that this person is good at figuring it out that way, and this person is -- That was just an added stressor, having to work in new groups every couple of weeks.

Some students commented that some initial time was needed to establish some familiarity among group members each time the instructor switched groups, giving students the opportunity to develop trust.

R: We basically said "Hi" and started working. We didn't really -- there was never really time to even talk about our anxieties or anything. There was just time to do the problem. So you need to structure some time so that people can get familiar with one another and to sort of trust each other, to figure out what they need to do, and -- It's really just that you can't just put people in there and say, "Work."

I: And expect them to work well together --

R: And expect them to understand that maybe some people -- maybe even ask, "Who has worked in a group and what has been your experience? What do you like about it? What are the limitations that you see?" And then you can work from there.

Some students explained that because it took time for relationships between students to form, switching groups too frequently inhibited the development of a supportive group environment. This person commented later four weeks was an optimal time to form a supportive environment in which students feel comfortable contributing to, and participating in, their groups.

R: I think that [in] this class you should try to work with other people, but when you are switching constantly, it doesn't allow for any build up of any understanding or trust or anything like that, and that is really tough to handle.

I: You think those concepts are important?

R: They really are. I would definitely switch groups, but I would perhaps do it a lot less.

Thus, provided there was adequate time to develop social relationships within their group, many students felt that group work provided a source of support in which they could discuss their anxieties and frustrations, and also assist one another in the problem solving process.

2. Students learn from each other through group interaction

Through the cooperative setting established in the groups, the students contributed their differing skills and knowledge. The interaction between the students generated a deeper understanding of the problems and their solutions. The students were also able to experience how their peers approached the problems, and to incorporate those ideas into their own strategies of problem solving. In addition, having a group of students to work with kept many of the students from getting overly frustrated when they were faced with difficult problems. They were able to explore different paths together, rather than struggling in isolation.

These are problems that you could figure out on your own but they would take you a lot longer than working together. And it just shows me I love group work just because you learn so much from other people. And you can totally learn a lot.

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The group discussion facilitates the whole problem solving method that we're going to use. As a group we go through all the different methods that we might use and try to nail it down to one that we feel might work and go from there. It's nice in that you're not an individual working on a problem. You're a group working on a problem. So it's just very different to have the different ideas bounce off one another....I know that just having discussions with others that we feel it's really nice to have support to have different ideas come together and form a stronger proof or theorem. And it's exciting.

Some students stated that learning the material through group work increased their understanding because their classmates could explain ideas to them at their own level. These students expressed that when students, rather than the instructor, explained ideas, it increased confidence for everyone in the group.

R: I think that [group work] reduces anxiety...I think if you're not all trying to outdo each other, then you are sort of able to give input to each other, also. And just by kind of knowing that other people are close to the same level as you in their understanding just makes you feel a little more confident about what you are doing. A professor that has studied it really intensively is you know, he's going, "Da, da, da, da, da", you know, my experience with other professors is just, "Da, da, da, da, da." And that's how it goes, and you're just like, "Oh, oh yeah."

I: In other words, "I don't really understand what you are saying."

R: Yeah, yeah.

I: But then, what's the difference when you're with a group?

R: Just that they are more on your level, and they can -- I think someone that is on your level is just more capable of bringing it to a level that you understand. And also it's just more comforting for me to know that I'm not the only one who's not sure. Or I'm the only one who is sure, and that feels kind of good too, you know. So it goes both ways.

This excerpt represents most students' views that group work can be affirming. Students felt that their peers could explain the material on a level appropriate to them and also share their own difficulties with one another.

3. Students learn that there are multiple ways to solve problems

Many students explained that through peer interaction in groups, they came to understand that there were multiple ways of solving problems. The problems in Math 130 encouraged a number of approaches, and through the group work the students realized, many of them for the first time, that different people think about math in different ways. This was important to students for several reasons. Many of the students' previous math teachers taught them to use specified procedures to solve certain types of problems and as such, provided little room for the students to develop their own approaches. With their new understanding about multiple ways of solving problems, some students felt more confident about finding their own approach to problem solving. In addition, students felt that this was applicable on a professional level because while teaching they would most likely encounter students who solved problems differently from themselves.

> The most important thing I have learned about math in this class is that there can be more than one right answer, especially with some of these problem-solving things. There is more than one way to look at it, whereas I was always under the impression that it was, "This is the answer. This is the way you go about doing it," and "This this this. The math is just this way." And you learn that there are many different viewpoints....I have always taken classes where this is the way you do it. Or even in calculus maybe there was another way, but both were taught to you: "You can do the problem this way or that." So I know, I have always thought of math as concrete. "This is the way it happens, so you have to do it." Just seeing, especially with problem solving, there are so many ways to go about it, and I've never really thought about math as being more open-ended and free. "OK, you can think of it this way and you can think of it this way, and you are going to come up with the same answer." And so definitely it is more the math class [that altered my view of math].
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> Sometimes [listening to other groups report their problems] gets frustrating just because if you can't figure out the problem and you're just sitting there and you're totally, totally, totally confused, and you can't figure it out, sometimes it's like saying, "How come they can figure it out?" and, I don't know, that gets to be frustrating. But it helps because then you see why you got the answer to the problem. But otherwise it's kind of interesting because some people solve problems differently. Some people graph out the whole thing and get the same answer, or some people do it all different ways, and it's just kind of interesting how people do the problems and get answers. It seems like everybody has something to contribute and so one individual's way of looking at something can greatly differ from another's point of view and neither is better than the other, it's just a different way of approaching it. And sometimes the other might have a way that seems more of a logical way of solving a problem, and so, "Oh, that's great. Let's go with this." And it's a chance to explain to the group as a whole versus, you know, I understand my way, but maybe their way, the other individual's way, is more logical to explain it to the group as a whole.

Thus, for many students, the group work introduced them to multiple ways of solving problems. This new knowledge redefined mathematics for some students, giving them an increased confidence to trust their own methods of problem-solving.

4. Group work engages students in the learning process

Many students in discussing Math 130 contrasted their experience in this course with the traditional lecture style learning of mathematics. One student commented that learning in this method was more efficient than the traditional method.

> R: Instead of sitting around and waiting for the material to be fed to us, we're doing the work in class. That's not to say we don't have homework, because we do, but it's interesting. I have this class and then I have another math class afterwards, and the other math class afterwards, we sit for the whole while and listen to the teacher explaining things, and I feel like, "Gee, this is such a waste of time. We could be doing the work right now and trying to figure it out, and then he could be explaining it after we tried enough." It's a lot more efficient use of everybody's time I think, the way we're doing it now.
>
> I: In 130?
>
> R: In math 130, it's more efficient because we're just sitting down and doing the work, and then getting help afterwards with what we can't do and getting explanations for a lot of things, for why the problems work out the way they do, as opposed to sitting in a classroom, bored, and then having to go home and do all the homework. It just seems like now that I'm used to this class that we're learning just as much, but it's a lot more efficient use of everybody's time, 'cause we're actually doing work instead of sitting there like zombies getting lectured to.

As demonstrated by this individual, most students expressed a distaste for sitting in a classroom in which the instructor lectured the entire time. Many students explained that through working in groups they felt more involved with the mathematics. The following two excerpts reflect this high level of involvement.

> It makes the time go a thousand times faster. That is one of the biggest problems in math courses; I just find a lecture about math incredibly boring; I can't handle it. Lectures are hard enough. For me, in my life, planning a career, it just gets to a point of "What am I doing here?" And then I just space out and get absolutely nothing out of it. In that way, I interact with a lot more of the problems because you can't sit in your group and not do anything. So it keeps me -- it is energizing for me to work in a group. Just kind of draw energy from the rest of the group. I am not going to fall asleep or daydream.

* * * * * * * *

> What we do in class is fun, time flies by. Sometimes I feel like I'm playing in class, but then when you realize the mathematical patterns and everything involved. I think it's just pretty neat, that it's so mathematical and fun at the same time.

In fact, some students became so engaged through working in their groups that they incorporated their new learning and understanding of mathematics into their everyday lives.

> We need to remember that we need to take turns because people get excited about answers. And you need to raise your hand once in a while just so people have a chance to speak up if they want to...It's very exciting when you feel like you've got it. It's that feeling like, "I did it!"...I definitely think about math more now than I have in the past, on a day-to-day basis...I think I've enjoyed math more this semester than I have in years.

Interview excerpts such as these demonstrate that many students in Math 130 are actively engaged in the learning process within the classroom and do not limit their involvement to the classroom context. Students attributed their engagement in the learning process and in the classroom setting to working in groups.

5. Group composition: The impact of the range in ability levels on group effectiveness

In this section we will discuss student views on the ways in which the varying levels of mathematical experience of students in Math 130 affected their working collaboratively in groups.

For many, varying levels of experience did not inhibit working collaboratively

Many of the students we interviewed explained that working successfully in a group depended upon the attitudes and beliefs of the group members rather than their degree of mathematical experience. Some students credited their group's success to the fact that all of the students were education majors and therefore had a similar orientation.

> Yeah, you're all in the same situation. Plus everyone in the class, as far as I know, wants to be a teacher. So, we get a lot of the same ideas, a lot of the same goals, and just a lot of the same background as far as motivation and stuff like that.

In addition, many students expressed that the most important factor in groups working cooperatively was for the group members to have positive attitudes toward collaborative learning. The following excerpt illustrates that for some students attitude was more important than ability for a group to work cooperatively.

> My experiences would vary on the groups that I had. I felt that you're going to have varying ability levels throughout the class. But, I found that the most prominent issue was not the ability level, but attitude. I've worked with some people, and you know I don't mean to be

> bragging or anything, but I've been in some groups where I just have more math skills that these people do. But, they've worked hard and that made the groups so much better and they ended up improving greatly and you know, ended up solving things that I couldn't solve. And we just worked together very nicely as groups. And then I had groups where the people did not care. Regardless of their ability level, the fact that they didn't care just -- the group didn't work. . . . Cooperative learning works only if it is cooperative.

Some students expressed that rather than hindering a group from working collaboratively, a range of mathematical experience within the groups actually benefited the students.

> I think that by having somebody that has good math ability in the group, it's good for them because they're learning to teach, and the people who are lower, it's good that they get the individual attention, because they have to do it. They can't just sluff through the whole time. Eventually they're going to have to learn it for the test.

For some, differing levels of experience hindered group effectiveness

In contrast, some of the students expressed concern that the range of abilities within the group prevented the groups from functioning effectively.

> If you're really comfortable in a group, that's important. That's a factor as far as how much you can work in the class or how much work you're going to get done. So if everyone's willing to contribute, then it works. Otherwise, if you get a group that's not so talkative and everyone's kind of shy, and math's not their strong subject, you kind of lag, then, behind other groups. Because there are definitely certain kids that are better at math than other kids, and there are some people in the class that haven't had much experience in math, or has forgotten everything--like myself, from high school, and you struggle a bit more.

Some of the students explained that when they were in a group in which the other individuals grasped the material more quickly than they did, they did not feel comfortable asking questions in the group.

> Sometimes I don't like how [the instructor] places us in groups....Because there's some groups I like to work with, and it's not because of the people or anything, it's just how I feel. Like when I go work in some groups, it's like they're at a different level than me. I know [the instructor] probably wants to do that, to get everybody from different levels together, but sometimes I just feel really intimidated, and I don't want to say anything.

Working in a group with individuals having more mathematical experience made some students feel self-conscious about their mathematical abilities. The result for some was to retreat from participating in, and contributing to, the group.

> If I don't understand something and everyone else has been through it, I will just write down the answer, because I don't feel like asking a thousand questions....This doesn't happen a

> lot; this isn't all I am doing in the class. But that logic stuff, I didn't really understand it. And they were going too fast. Basically, they were going too fast for me. And I got tired of saying, "Slow down. Slow down." Whereas if it was an individual assignment, I could have taken it home and worked through it. Maybe it would have taken me an hour where it took our group fifteen minutes. But I would have understood it because I would have done it myself and learned it.

Students who were consistently the passive recipients of knowledge in the group expressed that they did not benefit from working in groups. In the following excerpt a student commented on this problem.

> We work in groups, and we do our problems in groups, and we hand in our answer in groups. It seems like in our group, some people will just sit there and kind of listen and follow and I don't think that they really understood, but they didn't really want to say that they didn't understand. So, someone would explain it to them. But I think that when the test comes around, some people are going to have a hard time because they are just sort of used to listening and going on. Maybe they understand after it is explained to them, but they didn't get to figure it out for themselves.

When students were uncomfortable and intimidated by the abilities of others in their group they were more apt to sit quietly and be passive recipients rather than active participants in the problem solving process. As a result, these students viewed their experience in the classroom as far less effective than if the groups were working collaboratively.

In addition, some students who had more mathematical experience expressed frustration with the disparity in confidence and ability levels between group members.

> We switched groups and two of the people were very weak in the math department, and it was kind of frustrating that they would never speak out when we were throwing out ideas. They were kind of just silent and into themselves. There was just me and this other girl who were contributing, and they just kind of broke off by themselves. I thought that the point of the group work was to be contributing. So, even if you didn't have something to say, you could say, "Look at that, or what about this." So they weren't really saying anything, so it was kind of hard to facilitate a four-person group. We would kind of split off into two person groups. Then [the instructor] would come over and say, "Why don't you explain it to the rest of your group," and we usually were working on completely different things and when we showed it to them, they sometimes understood, and they sometimes didn't understand.

Some students perceived the varying levels of confidence and mathematical experience as preventing their groups from working cooperatively. Because of the primary emphasis on

group work in the class, these problems in group dynamics negatively affected some students' attitudes and experience in class.

Although some of the students attributed difficulties in their groups to problems of group composition, we came to realize that some of the problems were a result of the fact that the students were not used to working in groups and were adapting to a new classroom format. (See Tab 6, Section B.2. "Adapting to working in groups.")

B. Learning through the Problem Solving Process

In this section we will focus on students' perceptions of the problem solving process. Included in this discussion will be the student attitudes about the types of problems used in the course and since Math 130 is a learning environment in which students discover, rather than are given, the methods to solve the problems, we will discuss the ways in which students obtained answers to the problems. In addition, we will discuss how writing about mathematics affected student learning.

1. The problems and their role in the learning process

The problems assigned in Math 130 many times were open to interpretation and could be solved in different ways. In this section we will discuss the varying student views of the problems and how they affected the learning process.

The problems stimulated learning

Many students commented that the problems assigned helped them to increase their understanding of mathematics and to develop their problem solving skills. Many students felt that although the problems assigned were difficult, they were appropriate for their ability level.

> I: How do you feel about the problems you do in general?
>
> R: They're pretty good. I think they're accurate for ability, for what we're learning. They seem very appropriate for what we're doing.
>
> I: What do you mean appropriate?
>
> R: It's just they're not too high of level. They don't assume that we don't know anything and have no background in math. They're kind of like, "We know you have background in math, but we want you to look at it from a new point of view. So, we're going to give you a fairly easy problem but not one that anyone can solve. You've got to think about it, you've got to work it out. But it's not going to be the most complex problem you've ever seen either."

Many students we interviewed felt that the problems were an integral part of their learning in the course. They explained that the problems stimulated them to think about their understanding of

mathematical concepts.

> R: That's what a lot of the problems that we do ask us. To explain why we do this or why it works that way. And I think that we need to understand that in order to teach children about it.
>
> I: What would you consider the most useful part of the course?
>
> R: I think the problems [the instructor] gives us are very thought-provoking in that they really make us question the methods we use to solve them. And I think that is really helpful because I think giving just a set of problems, like normal equation problems aren't helpful and I think these kind of help us think about [our understanding] better.

In addition, the nature of the problems facilitated an exploration and deeper understanding of the problem solving process for some students. The following student, a former engineering student who had taken a year of calculus, viewed the problems as a challenge to use his/her logic and knowledge about mathematics.

> You could open the workbook and say, "Well, these are really simple problems," but if you think about it, these problems involve a lot of logical thinking. I think I learned a lot of that with engineering. It's a very logical field, [an] extremely logical, analytical kind of field. And so I really like this class because the whole logic thing comes really easy to me. To be able to pick things apart and to be able to speak about them with a certain process or in a certain way and...I really like being able to do that. You know, some of the problems are actually quite hard and it's kind of interesting because you have to think, "Ok, how do I solve this problem? Ok, well you could do it this way. Or if I did it this way will I get the same answer or will I get a different answer?" And that's just really interesting to me.

Problems assigned at the end of the course were more dependent on prior mathematical knowledge

Some students commented that the problems that were assigned in the latter part of the course depended upon more on the students' mathematical background more than the earlier problems.

> R1: I prefer where everybody [in the group] talks, but that was kind of more at the beginning, where the problems were a little easier.
>
> I: So, are they more difficult now?

R1: For me, yeah. Well, I'm thinking that the test is going to be harder. I'm hoping that [the instructor] is going to start giving us a little more guidance, because I haven't had algebra in a while.

R2: Yeah, that would definitely be harder for people who haven't had that in class, 'cause we're getting into ---.

I: You're getting into what?

R2: Just fractions and algebra and all that kind of stuff, not just an abstract concept that no one in the class has ever seen. Like, for the last class with truth tables and stuff. But now it's algebra. So, for the people who've had it recently, it's more like just figuring out the equations for the word problem. But the people who haven't had it, not only do they have to figure out the word problems, they have to figure out just an equation in general 'cause they haven't had it in a long time.

As these students noted, the nature of these problems made some students feel that they needed more input and guidance from the instructor. Because the later problems relied on background knowledge, many of the students we interviewed commented that differences in the students' mathematical background were a source of tension and frustration in the classroom. The following student expressed concern that problems which assume some mathematical background created conflicts in some groups.

I mean this is something I've noticed since spring break too. It's like some of the groups -- we're working on more complex things. Before it was more -- I want to say simpler math, but I guess it was kind of more basic math before. And now we're working on probability and algebra and before we were working on truth tables and stuff. It just seems to me that the content of the stuff is getting a little bit more difficult. So, I think some of those differences between student levels are starting to show a little bit more than they did before. And some groups -- I've noticed some people that really know what they're doing get a little agitated with the people who don't know what they're doing instead of listening to understand what they're saying.

2. The process of coming to closure: finding solutions

It is important to note that the learning process in Math 130 differed from the traditional lecture-based math courses in which an instructor provides the students with the method of solving the problems prior to the students themselves attempting to solve the problems. In Math 130, students were assigned problems with little or no introduction by the instructor. As students worked in their groups, the instructor acted as a guide in the problem solving process without directly providing the answer. Since the students were not initially provided with the methods with which to solve the problem, in this section we will discuss student views on finding the solutions in this new learning format.

Interviews with the students indicated that although many students were initially frustrated when they weren't provided with the method for solving the problems, most students were able to obtain the answer through working in their groups on almost every problem as the semester progressed.

> So, I mean there's times that I think, we at first were really frustrated because it was like, "What's the answer? What's the answer?" And that was at the very very beginning. And that was like class number two or something. And then we learned what the structure of the course was going to be and then we actually did find out what the answer was to that problem at the next class. And then at the next group discussion, the next class discussion that met two days later, we figured [it] out and then we were satisfied with it and then we moved on to the next problem.

* * * * * * * *

> I: Have you ever not been able to solve a problem in your group?
>
> R: I don't think so. You know, if anything, we'll get the answer. This has happened a couple of times, we'll figure out the answer, but someone in the group -- we'll try to explain it to them but the bell will ring and then like one person might walk away feeling like they didn't understand. That's happened just a couple times but, I think every time we've solved the problems, yeah. The class definitely does not move at a really fast pace. It's not like time pressured really, so that's good. [The instructor] gives us more than adequate time to really work the problem so that everyone really does understand and explain everything so that's good.

In all sections, when students were unable solve a problem in their groups, most felt confident that they would at some point obtain a solution using the resources available in the course. Most students obtained the answer in the class discussion, mainly through other students or else through hints by the instructor. In some sections, if the students could not find the answer as a class, the instructor would provide the solution and method in the class discussion.

> I: What happens when you don't [get a solution in your group]?
>
> R: If another group finds the answer then they would present their answer at the end of class, so we can get it through another group. There have been occasions where nobody gets it, where [the instructor] will give some assistance. So, if we don't get it, somebody is going to get it and we eventually see it.
>
> I: So you always know that at some point you're going to get the answer?
>
> R: At some point.

In other sections, when the students were unable to find the answer and method as a class, the instructor would provide hints, but refused to provide the solution. In these cases, the instructor would move on without solving the problem. Students were then left to work on the problem

outside of class and see the instructor individually, if necessary, for more help.

> R: We move on without answering all of the questions. And that's because [the instructor] wants us to spend time on our own to figure out possibly what the links are. Sometimes you know all the answers and it might be that the class hasn't necessarily told you, or [the instructor] hasn't told you what the answers are. But you might for yourself feel satisfied that you know the answer and that's good and then you're set to write-up. If you have questions or you're not sure after reviewing the problem on your own then you're welcome to go see [the instructor] and ask any questions. And again, [s/he] won't outwardly say, "The answer is this." [S/he] will allow you to find out what the answer is by just talking to you and asking certain questions in such a way that you can figure it out for yourself.
>
> I: So you generally feel satisfied with the questions?
>
> R: Not always just when you leave class but you know that there's a way to find the answer and it's up to you to get there. To go to the teacher to find out what it is. Or to talk to another classmate or just talk to your group.

In summary, most students felt confident that enough resources were available in Math 130 to find the solution to, and method behind, the problems. For a more detailed discussion on the amount of help the instructors provided in the problem solving process, please refer to Tab 7, Section C.2 ("Transferring responsibility for student learning"). In addition, an analysis of student views on the amount of help and support given in the problem solving process appears in Tab 6, Section B.1 ("Adapting to the 'Instructor as Guide' teaching process").

3. Learning through mathematical writing

For most of the students we interviewed, the process of writing up the solutions to problems was a critical part of learning the material. Many of the students felt that in doing write-ups, they realized whether or not they had actually understood the problem. They also found that writing down solutions helped them to clarify their thoughts.

> You had to have solved the problem pretty much before you started the write-up. And then you had to take how you solved it and put it into words and explain it so that someone who had never heard of math before could understand it. So that was definitely tricky because you had to not only figure out how to solve a problem, but figure out why you solved it. And that was a lot more than other math classes had ever done for me.
>
> * * * * * * * *
>
> [The write-ups] help me understand. They make me think through a lot of stuff a lot more than I normally would if I didn't have to write things up.
>
> * * * * * * * *

> [I think the write-ups are helpful] because when you have to write it down, it makes you think about it a lot more. You know, you have to explain it, especially writing. At first I was very frustrated because it's very hard to explain math in words, but I think it helps a lot. I really do.

Many students expressed that the write-ups were an integral part of the learning process in this course and complemented the group problem solving. Through the mathematical writing the students sifted through and came to understand the material learned in the group, in their own individual way. The following interview excerpt illustrates the individual learning process that occurs through mathematical writing.

> A lot of time things like snapped for me as I was actually writing them because I was like, "Oh, that's why this happened," and then I would become more involved and it was more a detailed assessment of actually what I learned. I wasn't just providing an answer. I could get that from the back of the book. I could look at somebody else's paper. But what I was providing my understanding of what the problem was.

A further result of the mathematical writing was that it assisted some students in making a shift from focusing exclusively on finding the answer to understanding the process of problem solving. In our interviews students stressed that the point of the write-ups was to explore why a particular solution worked instead of just giving the answer.

> In a sense you are solving [the problem] and then you are going back and you are almost re-solving it by writing down how you came up with the answer. So it answers that "Why?" that I think that people are coming across. It's not just, "This is it." It's, "This is it and this is why."

* * * * * * * *

> I think writing something just clarifies whether you really solved the problem or not, or understood the problem. I like it. At first it was difficult to write about math but it's good in that I think it will help us explain ourselves when we're in front of the classroom. We have to explain why.

While many students valued both the individual and the group write-ups, some students expressed concern over certain aspects of the group write-ups. One such concern was that it did not allow for the individual component in the learning process. One student stated that,

> There's a couple reasons [why I don't like group write-ups]. I think one is it's very difficult to meet in a group outside of class....most of us have a couple jobs, and you know, a lot of other classes....And I also think the time that I spend at home doing the write up on my own is sort of essential in my understanding of the concept....And when you work it through in a group, you kind of just work it through in a group almost like you do it in class, and you have a general idea of what you are going to say, but you don't have the specifics. And if, you know, I'm not the person doing the write-up, then I don't feel as clear on what I have learned.

In addition, some of the students expressed concern that, in the group write-ups, their grades depended on how well another person represented the ideas of the entire group.

> We got a problem that in my group there was one person who basically took over the whole thing and it was really annoying, and I didn't have very much say in the group write-up at all. But I think I basically sat back and I went, "OK." So I basically had to take the grade on the write-up without ever having a say in anything.

In summary, the write-ups that the students did after some of the problem sets were an integral part of the course. For many students the ideas that they encountered in their small groups were clarified and stretched further through the process of explaining their solutions in writing. In addition, the process of writing helped many students to become aware of what areas they didn't fully understand and to focus on developing those parts. Finally, in writing their solutions the students practiced explaining how to solve problems. Many students particularly valued learning how to explain their solutions because the written explanations, as well as the verbal ones, helped to prepare them for teaching.

C. Student Views on Assessment

Students, when faced with a new course format in which they worked exclusively in groups with little or no lecture from the instructor, expressed concern over how they would be assessed. For many students, grades were a strong motivating factor and they were anxious to learn how they would be assessed in order to succeed in this new classroom format. In this section we will discuss students views on the methods of assessment used in Math 130, particularly with respect to the exams and write-ups.

1. Group-based versus individual exams

Many students were surprised and a little anxious that the testing methods were similar to those used in a traditional lecture class, feeling that the assessment strategies should have been modified to fit the new class format that focused on collaborative learning without a lecture. Most students we interviewed expressed concern at the beginning of the course that the tests were to be taken individually, explaining that since most of the work was done in groups, they should be tested in groups as well. The following student's comments reflect many students' reactions to being tested individually on the midterm and final.

> Something that concerns me is that I asked [the instructor] on the first day of the class that I wasn't really sure how the tests were going to be handled because I thought the group work, if we're so dependent on it -- not that we're totally not thinking for ourselves, but you know we get a lot of the answers off of each other. And I asked [the instructor] about that. I said, "I'm kind of concerned. What's the format of the test? Are we going to get in our groups and go through it or what?" [The instructor] said that we're going to be learning specific ways to master problems and those ways we're going to be applying to the problems we have on the tests.

As the course progressed, student opinions about individual testing differed. While some disliked the individual aspect of the exams throughout the semester, others considered it a good way of balancing between group work and independent work. For many students, this concern about individual testing was alleviated once they realized they were actually learning in their groups the material that was necessary to do well on the exams. The following excerpt is representative of the feelings of many students in Math 130.

> R: Yeah, at the beginning I was just kind of confused as to how exactly this was going to work, because [the instructor] told us that the assignments would be done in groups, but the tests would be taken individually. That was a big concern of mine. Because I didn't understand -- because we were bouncing ideas off other people and we relied on other people at certain times, how we were able to do the test individually? That kind of bothered me. But at the end of the semester it was possible to take a test individually, even though we'd been working in groups throughout the semester, and so that's something that I might have realized, I guess.
>
> I: What made you realize that?
>
> R: Well, when I took the test, and when [the instructor] gave me the review sheet, I guess, everything just clicked through working in those groups. So I got to the point, where I didn't need my group there anymore to help me figure out the problem. I could do it on my own.

Some students liked the individual exams and commented that individual tests and write-ups were necessary to both balance the group work and assess who understood the material.

> I feel like it's more relevant for us to be doing [the exams] on our own and writing-up our own, 'cause we work in the groups already and teach each other that way, and then that gives us ideas on where to take it on our own. So I feel like we already do enough group stuff that we should then remove ourselves from that group and use it and see what we do with it. And that's how we should be graded. I don't think it should be a group grade.

However, some students disliked the individual testing throughout the semester, stating that the group exams would better reflect the focus on group work in the course.

> The only thing I don't like though is we took our test individually which was weird because we've never worked on anything individually. We've always been in groups and relied on the other people in our group and then we were thrown into a test situation and then we were on our own. And that was kind of weird, for me, anyway. [The instructor] said that it wouldn't be fair to mix some groups with other groups because some of the groups are doing a lot better than other groups. But, I didn't like being thrown in on it alone.

Some students commented that if they were to be tested individually, they should be given the opportunity to work outside of the group in preparation for the exams. Others felt that there should be some balance between testing individually and as a group.

> I: If you could invent some sort of ideal set-up for the class, including this method of testing, what would you do? It doesn't have to be something that's already in existence, but would you change things?
>
> R1: I would keep the group work. Maybe for a test, if you want to take the test as a group, maybe I'd take one test with the group and then one test individually.
>
> I: Maybe a midterm and a final?
>
> R1: Yeah, like that. Take one with a group and then take one by yourself.
>
> R2: I think it might be helpful even just to do more individual work. I know it's hard and it would be, I would hate it, but it would really make us start having to be able to work out problems by ourselves.

Some students continued to feel throughout the course that the primary emphasis on group work prevented them from being able to fully understand problems individually. This was reflected in their desire for group exams.

2. Concerns about grading procedures

Since the students had little or no experience with explaining mathematics through writing, many were unsure of what was expected of them in order to do well in the write-ups. Many students commented that they were unsure what the instructor wanted in the write-ups and stated that an example of an good write-up would have been helpful. The following student stated that discussing such an example in class would reduce some of the students' frustrations with the course.

> A lot of people are pretty vocal about grading for the course. They really don't understand how it is handled, and what is going on there. I don't really have a problem with it because it has worked pretty well, but I'm sure that if I got a poor score that I might have had a problem with it. But I think that it would have been good to -- we did a couple of assignments and [the instructor] went over them as if they would have been graded, but they did that outside of class. I think that it would have been really valuable to do an assignment and go over in class and say, "This is what we want. This is why this is wrong. This is why this is right." Practice assignments were helpful, but they would have been more helpful if we would have gone over them together in class. So people would just have a more solid understanding, because up until now, they have never seen the papers

being graded. We have got these notes on why something is wrong, or why something is right, but we have never seen the process actually done. And I think that the people would be a lot less antagonistic if they would know what was going on a little better.

Many students expressed concern that the grade they received on the write-ups did not always reflect their knowledge and understanding of the problem. Many wanted to know how the instructor determined grades on exams and write ups and suggested that the instructor provide specific examples of "A," "B," and "C" level work.

[The write ups] were worth ten. And then we got so many points out of the ten. But it was never clear. I mean half of you got for just doing the problems. But it was never really clear what seven out of ten was or nine out of ten was. Because if you're going on a traditional percentage grade seven out of ten is kind of a lower grade....But some people got really low ones because they didn't really figure out how to do the problem or how to do certain parts of it. And they were really frustrated you know, because they were getting lower scores and also the fact that not ever really having been told what goes into the write up. So I guess I'm just unsure. I just think it should be addressed a little bit more. You know, "What is good?"

D. Applications to Teaching

As preservice teachers, the students in Math 130 viewed the method and the material learned in this course through the lens of applicability to teaching. Many of the students we interviewed stressed that this course had provided them usable skills that they would eventually apply in their own classrooms. When students criticized the course it was often because they found that certain areas were not applicable to teaching.

1. Math 130 models a teaching method

Some of the students found that the method of teaching presented in Math 130 was effective both for learning mathematics and for modelling a way of teaching that they could apply to their own classroom. Some of the students felt that one reason the course was required was precisely so that they could learn this method of teaching. In the following two interview excerpts each of the students discuss their perspective on the applications of what they have learned in this class.

I think the purpose of [the course] is to look at math in a different way. To understand math, to approach it from a different point of view. To bring into a classroom a situation that facilitates group discussions and arguments and solutions. And I think it's going to be a really effective tool that I can use as a teacher to kind of get back to the basics, to get back to the foundation of math. To have them go through the learning process themselves rather than having me explain it to them. Because I don't think that's how children or individuals, children or adults, learn. I think you need to work it hands-on and you tend to remember more. You understand more about it, rather than having me stand up in front of the class and say, "This is what you do, so do it," and they do it.

* * * * * * * *

I: Would this class have an impact on how you plan to teach?

R: Yes. Knowing where certain things come from and why. To teach base ten and all this stuff that we have done. Why we teach what we teach. That would help me teach. I would want to teach more like how [the instructor] teaches, where the class is learning and not telling people the answers. Because like [s/he] said, once you tell the people the answers, no more learning goes on. It's over. [The instructor] did it one day on the board. Everybody just got really quiet and nobody had anything to say because it was over.

One of the ways in which students found that this course modelled a method of teaching and learning was in the area of discovering multiple approaches and multiple solutions in the problem solving process. The students found this to be particularly powerful, in part because of the applicability of this concept to their teaching. They realized that when they eventually taught, their own students would solve problems in different ways. This was confirmed by our discussions with some of the alumni of the new method who were teaching. They did find that learning about multiple strategies for problem solving was directly applicable in their classrooms. In the following two interview excerpts, student teachers explain how they used information from Math 120 classes (that used this method) in their own teaching.

I: Do you use anything from Math 120 when you are working with the kids in math?

R: For the things we are doing, this is the beginning of the year. I am working with four-year olds. So certainly in terms of problem solving when we are sitting in group and there is something presented. If a child gives an answer, it is a non-judgmental response and then we go on from there and say, "Is there any other way you could have done this?" We just keep brainstorming different ways the problem could have been solved using a lot of different strategies, encouraging the notion that there is no one right way to do anything. Different strategies work for different people.

I: Is that something that came out of 120?

R: Sure. And it came out of 120. With a working model in 120. Certainly in my theories class it was presented, but to then experience 120 as a working model was great.

* * * * * * * *

I: Were there any parts of the course that you found useful?

R: That I found useful? Yeah, sure, knowing that there's more than one way to solve a math problem, and learning that if you can't get it on your own, that it's okay to ask someone else for some help...

I: Do you use any of the information, or do you use anything from 120, now that you're teaching?

R: Um, just a little bit. I mean, I had one child come up to the board and show me how he

> solved the problem. And then I hear another student say, "Well, I solved it a different way." So I had that student come up and show that too. And I would say that both answers are correct and you went about it differently, and that's fine.

The importance and the value of experiencing this course as a model for learning and teaching was confirmed by our interviews with alumni of Math 120 who did not experience learning math in this way. The students who learned the old method of Math 120, in a predominantly lecture mode, saw a sharp division between how they learned to teach in their Math Methods course and the way they were taught in Math 120. Some of these students, particularly the student teachers, felt that the university ought to use the math courses that they are required to take as a way of modelling how they should teach.

> Well I just think that if the university is going to have methods courses that really promote CGI and really promote problem solving and looking at math in a whole new way, it should also be carried out in other classes that are going on in the university. It's kind of like a practice what you preach kind of thing to me...Because after [Math 120] you take the CGI class and you're supposed to teach that way and you've never even been taught that way so it's kind of hard to see it, you know, what it would be like.

For further discussion on the relationship between Math 120 and teaching, see Tab 5, Section E ("Alumni").

2. Learning how to explain mathematics

Many students discussed how this course helped them to learn how to explain mathematics and that this was directly applicable to their future teaching. The students found that this learning occurred both through the group work as well as in the process of writing the solutions to the problems. The following interview excerpt illustrates both the process by which students learn to explain through group work in this class, as well as the applicability of this knowledge to teaching.

> ...I know that later on, I'll take a curriculum class about how to teach math, but in a way, [Math 130] almost is a curriculum class just because you are learning how, I've learned a lot about how to explain math to different levels of people....you are sitting in a group, and not everyone in your group is at the same, I don't think any of us are math idiots, but like sitting in this group, some people understand math concepts better than others. So, trying to explain to someone who doesn't get algebra as much as they can do something with, like, geometry. You have to kind of like explain it in their terms, like learn how, so you are kind of learning different ways to explain things to people who explain things in different ways.

Another student discussed how the mathematical writing in Math 130 was applicable to teaching in the sense that it helped her to explain at a very basic and simple level.

> I just really think that the write-ups are important to kind of learning how to explain. And [the instructor] puts a lot of emphasis on you being clear and precise, and that's part of the

> grade. And I think that that is also essential....And I think, "OK, so how would I explain it?" So I really think that the write-ups are essential to kind of gaining that understanding of how you're going to go about explaining it. And even though some of the math concepts are more difficult than I would ever be explaining to a first grader, I think it is still essential to being able to think down to the lower, more simple level of understanding.

Some students highlighted the applicability of the concepts or processes learned in this class particularly when they thought that the content was likely to appear in the classes that they would eventually teach.

> [One of the write-ups was] about algebraic algorithms, like, explaining algorithms - addition, subtraction, multiplication, and division, the four basic ones that they asked. And those, I thought that was a good idea 'cause it wasn't, like, exceedingly difficult, but you got to places where you could say, "Well, here's a place where somebody might not understand this," and then you could think about why you understand it and how you'd explain it to them. So that those writings did some good.

Some students expressed concern over how these concepts would relate to their experience as future teachers, and wanted the instructor to provide information on translating these concepts to the classroom.

> R: I think it would be better if they followed, if we were given problems and then they followed up by giving us, like sources or something that we could use in the classroom, so that we could say, "Okay, this is how I was thinking about this problem. Now, how might I change this so that I could do this for my third graders or, how did this relate?" I don't think that there is enough connection of how we are going to use it later.
>
> I: So when you say sources, you mean texts or any sort of reference?
>
> R: Or problems, or workbooks, or just examples of problems. If after each problem that we can see that as a class there was an example of a similar type of problem that we could use with various grades. I think that would be great. I think that would be tremendously helpful, and it would make things a lot clearer to these teachers. I don't think that they understand where the connection is.

Some students were particularly critical of aspects of the course when they could not understand the applicability of the material to their teaching.

> A lot of this stuff that we do in the class I feel like I'm never going to use because I'm planning on teaching such a young age that I don't need to do it. So I get sick of writing them, because I feel like I don't need it....There were only about three write ups that I can say that I'll ever actually need and care about....My kindergarten kids aren't going to ask me, "Is the square root of two a rational number?"

* * * * * * * *

OK, there was one assignment where we had...to explain how we would teach a class, how we would explain base 4 or something. I just felt like these kids barely understand base 10, why would we teach them base 4? You know. (laughing) I just felt that was pretty impractical. I know that I would never even dream of doing that with my class. They would have no idea!!

E. Alumni

As mentioned in Tab 3, Section B.3 ("Research questions and methods"), we interviewed thirteen students who had taken Math 120 under the old method and seven students who had taken Math 120 under the new method. Of these, eight of the old method students and three of the new method students were currently teaching or student teaching. Due to the small number of students, we cannot draw general conclusions about the views of all alumni. However, we can suggest what patterns emerged.

Interviewing the alumni of Math 120 provided us with valuable information on which aspects of their experience in the course were significant after the course had ended. In particular, the alumni who were currently teaching were able to understand the relevance and applicability of Math 120 in an actual classroom situation.

Alumni of the New Method

The alumni who were teaching or student teaching emphasized the applicability of the course to teaching. Two of the most important things that the alumni had gained from Math 120 were a deeper understanding of math and a student-centered approach to teaching with a focus on problem solving. Two of the three alumni who were in a classroom also drew parallels between the new Math 120 and their math methods course. One of the alumni with preK-3 certification described how Math 120 presented a working model for the information that she learned in her math methods course:

I: Do you use anything from Math 120 when you are working with the kids in math?

R: For the things we are doing, this is the beginning of the year. I am working with four-year olds. So certainly in terms of problem solving when we are sitting in group and there is something presented. If a child gives an answer, it is a non-judgmental response and then we go on from there and say, "Is there any other way you could have done this?" We just keep brainstorming different ways the problem could have been

solved using a lot of different strategies, encouraging the notion that there is no one right way to do anything. Different strategies work for different people.

I: Is that something that came out of 120?

R: Sure. And it came out of 120. With a working model in 120. Certainly in my theories class it was presented, but to then experience 120 as a working model was great.

It is important to note here that the alumnus was using the methods in the course to teach preschoolers. This is significant because some of the current students felt that Math 130 was only relevant to the older grades and that students planning on teaching younger grades would never use the material presented in the course. This alumnus described how for him/her even if the material is different, the strategies could be applied to younger grades.

The following alumnus also described how Math 120 was the first class that implemented the kinds of philosophy that are being taught in the school of education.

It was an active class, which is good. I think most classes should be like that. I personally learn better that way; I think most people do. So in that way, it was great, that kind of philosophy that they were telling us that you need to teach, you know the kind of stuff we have to have when we're teaching is being implemented for us. And that was the first time that I had really experienced that before.

The same student felt that there were a lot of course requirements that were redundant. S/he felt that the math courses and the math methods course could be combined into a single course which deliberately taught the students mathematics using techniques which they could apply in their own classes. In the following interview excerpt the alum described how s/he would consolidate Math 120 and the methods course:

I think you could spend a class period discussing one of the types of group problems, having to go working on a problem like that. [The Math 120 classes] are 2 hour classes I think, so spend the first hour doing that, and spend the second hour discussing - then spend the next half hour why that problem might have been frustrating, And then the last half hour applying it towards teaching or something like that.

Alumni of the Old Method

To remind the reader, the old method refers to any approach teaching Math 120 other than the new method. It is important to note that while there were some similarities among the old method sections as taught by different instructors, there was no single old method as such. In addition, some elements of the new method, such as the use of the large group discussion, appeared in some sections of Math 120 using the old method.

In general, the students did not like the exclusive use of the lecture method of teaching regardless

whether it included going over homework. One exception was a student whose instructor used a lot of drawings to understand problems. This student had experienced difficulties in previous mathematics courses, but was able to understand the instructor's explanations and left the course feeling that math was not so much of a mystery.

In contrast to student views on a lecture-based method of teaching, the students really enjoyed the small group and large group interactions that occurred in two of the classes. One of the students, whose instructor had switched to using small groups at the end of the semester, discussed how much s/he preferred the group work.

This student also explained that the classrooms today, unlike the classes when s/he was going to school, use a lot of cooperative learning and s/he felt that the instructors should be aware of that fact:

> R: I wish there would be special courses that are in pre-school through third. I feel like some of the course work that I have had has been so high that I don't understand how to apply it to my certification level at all. I think that was one of the biggest complaints that I have heard from colleagues. "Why am I learning how to do fractions if we don't do fractions?" I think that professors who are teaching these courses need to be in tune with what is being taught in schools and how students are being taught in schools today. What are goals of how math is being taught in school is and then applying that to their teachings for the university students to be able to learn.
>
> I: How did you feel that 120 measured up to those standards?
>
> R: I don't think it met it at all. I think that at the very end it was starting to and I really wish the whole school year could have been like that.

This student also expressed confusion over the purpose of the course:

> I guess I wonder what the focus of the class is. Is it teaching teachers how to teach math or is it teaching teachers how to understand math? I don't think I saw that focus in 120 until the end of the year when we were doing how to solve problems differently and doing the games and stuff....The end of the year was more things that I could see in my classroom, which was solving problems differently or doing games hands-on in small groups. It was seeing that the material [the instructor] was giving us could be used in the classroom. I guess I really didn't see that in the beginning.

The class which used a large group discussion format had a significant impact on several of the students. In the following excerpt a student discusses this experience.

> R: For me, I hated math growing up and had a really hard time with it in high school and decided to take the two math classes here because I wanted to be a better teacher. I didn't want to go into teaching it to my kids not liking it, and it was the best thing I could have

done because both classes helped me so much to overcome my fear of math and understand it. And now I mean, it's fascinating....[Math 120 and Math 122] explained things in ways that I've never seen. I don't know. It's just changed my ideas of, you know, "You can get this. It is easy to understand."....And it gave me a boost like on the first exam, in [the instructor's] class, I think a got like a 98 and I'm like "Hey! I can do this!! I'm good at math." And that was, to get that positive reinforcement was really good for me. It really helped me. Because I thought, "I'm smart. I can do this. I know I can get this." And so, from then on, it was a piece of cake once that turned around....

I: And so in what way was Math 120 different?

R: It just started over from the basics and explained things, explained numbers. Like, why a certain number is always divisible by 3 and how you can tell. I mean, it just made things that we'd always been taught all along, but never explained why, kind of like CGI math....

I: What was your favorite part of the course?

R: I don't know. I liked it all. I just enjoyed being able to understand, like get an answer and have it be right, and be able to explain how I got the answer. You know, and have it make sense. To me that was really important.

I: Have you used that in your teaching at all now? Those kinds of things.

R: Yeah, we always -- whenever I'm lecturing, and well, not lecturing, talking to the kids about a math problem and they put an answer down. I always ask, "Well, how did you get that? And why did you put that answer? Why didn't you put this one?" And we'll talk about it as a whole class. "Is this right? What did you get? What would you do differently?" I think that's really important for the kids. To let them know, "OK, this is the wrong answer, but your thinking of getting the right answer was a good way." So it's not -- the answer is wrong, but your thinking is not wrong. That you're not thinking bad; you just have to think a different way. I think that's important.

Another student explained that his/her favorite part of the course was the story problems and sharing because, "I think it was the first time that math ever came from me." This particular student had done well in math before, but without fully understanding what s/he was doing.

I never really had to think about math before I got into college. It was all memorize this algorithm and then spit it back to me, and I was really good at memorizing and spitting it back. I could get really good grades at math without really understanding....The way [Math 120] was taught was that we were given a problem and we couldn't use any of our algorithms or geometry or algebra or anything to solve the problem. We had to think about it like a child would without any of those crutches. And we had to draw pictures. It was just incredible for me because I never had to do any of that before or think about math before...Now when I teach math, I am always asking, "Did somebody solve this a different

way?" and then sharing a different way they could solve it in case they didn't understand it that way.

Almost all of the student teachers talked about group work and its impact on understanding multiple ways of solving problems. The students who had not experienced this in Math 120 learned about it from their methods course or their cooperating teacher. Many of the students discussed the methods course in conjunction with Math 120. A student who had worked in small groups during the latter part of the Math 120 course said that because it incorporated parts of the methods class it was "practicing what you preached." Other students wished that their course had used group work. A number of individuals were angry that their math course was <u>not</u> like their methods course. They felt like the university should teach as they were expecting their students to teach.

From the Researcher's Point of View:
Analytic Generalizations about the Student Learning Process

The students taking the Math 130 course were faced with a completely new format for a mathematics class. They were accustomed to learning course material from a text and from the instructor, predominantly through lectures. Their expectation was that the instructor would provide them with background knowledge, information about how to solve problems, and feedback about what material they should learn and if they were learning it appropriately. By contrast, in this course they were expected to learn material predominantly with other students and on their own.

It is important to note that interviews and observations of the classes revealed that varying levels of feedback were provided in the different sections. In some sections the students received little or no help in the problem solving process and had to determine for themselves whether they were on the right track. In others, students were given more guidance in the form of definitions and examples as well as feedback on whether they were on the right track. For more information, please refer to Tab 7, Section C.2 ("Transferring responsibility for student learning").

Though students received different degrees of help in the various sections, two patterns emerged in our interviews: many students experienced a shift in the source of knowledge in this new course format and some experienced difficulties in adapting to this new format.
In this section, we will discuss these patterns.

A. Shift from Instructor-Centered to Student-Centered Culture of Learning

As students worked on problems in groups, they were unable to rely on the traditional sources of knowledge such as instructor or a text. Thus, students needed to look for other sources of knowledge in order to find the solutions. For many students, this resulted in a shift in the source of knowledge from the instructor to themselves. In this section, we will discuss this shift and its effect on students and their relationship to learning mathematics.

1. Shift in the source of knowledge: change in instructor role

In Math 130, most students indicated that the role of their instructor was not to directly teach them the mathematics but to facilitate their learning through the problem solving process. The following excerpts are representative of how Math 130 students viewed their instructors' role.

> It's more student-oriented I guess, it's not teacher-oriented really. [The instructor] doesn't lead the class basically...

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> [The instructor] is more like a "guide on the side," as they would say. [The instructor] will say, "Get into groups. We're going to work on this problem."...It's very rare that [the instructor] will go up to the board and teach something.
>
> * * * * * * * *
>
> I: What's the role of the instructor?
>
> R: [The instructor] is actually there just to facilitate our discussion, make sure we're on the right track. [The instructor] is there to provide counter arguments. If we feel we've solved something [the instructor] might bring up something that we've maybe overlooked, which we need to go back and look at again. Or to walk through the problem with us and as far as trying to be an outsider. Saying, "Okay, this is how you suggest that someone solve this problem, if I use this, is this the answer." And [the instructor] would either agree and say, "Alright sounds good," and then [s/he] would move on to the next group. [The instructor] is not there to provide answers. Just to give instruction. Because I think [the instructor] feels that we know, [s/he] knows that we're capable of giving the answer. It's just a matter of when, and just working on [the problems] a little bit more.

In addition to this shift in the instructor's role away from directly teaching the material, this was the first time that many students had not had a text in a mathematics course. For virtually all of the students it was very easy to adapt to not having a text. In fact, many students expressed a preference for not using a text.

> Math books bother me [Laughter] because they have so many problems or at least the ones I've had are just problem after problem different words or different numbers but they're just repetitious. Once you get the idea and the types they just make you exhibit the problem with different numbers. And that seems like at least every other day if not everyday, it's a new different problem. It may have- like their sections in our folder like Algebra or fractions or whatever and we'll stay in that section for a while but they're completely different problems with different ideas.

A few students indicated that at first they were anxious about not having a text, but as the course progressed, they became comfortable with the absence. In the following quote, a student expressed the shift in his/her perceptions of the need for a text.

> At the beginning, actually, I went to the book store to buy a book, and it's like, "There's no text! How can there be no text, what do I do?" That's 'cause I was anxious about it to begin with....As far as for this course, I don't think we needed a text. I think that when we needed information we were given the information, and as long as you took notes, or if you were there in class you were able to get it. In a personal sense, I think it's maybe 'cause I haven't had a math class in awhile. It sort of re-sparked my interest in math, and I think it might have helped to have a text book so that I could adhere to it and refresh some of the things that were unclear to me, or to confirm what I was pretty sure was true, but wasn't positively sure. I don't think it was needed for the course, but maybe it would have helped some students, but I think maybe it also would have kind of cut back on the process of

> figuring things out. They'd always have this text book to confirm that they're right, and maybe the confidence that you got from just figuring it out might not have been as strong.

This student is experiencing the shift in the source of knowledge that marks this new course culture. S/he still feels more comfortable relying on an external resource, but simultaneously recognizes that the "process of figuring things out" is valuable and might be hindered through the use of a text.

The absence of a textbook in conjunction with the shift in the instructor's role away from directly teaching the material resulted in a shift in the source of knowledge for the students. Rather than relying on an authoritative person or text for the mathematical knowledge, students turned to other sources.

2. The change in student role as the source of knowledge is shifted

The new role of the instructor in this class provided the opportunity for students to learn the material in a new way. The process of learning math for the students in this class was predominantly group-oriented. Most students turned to their groups as a resource and in fact, preferred working with their groups to learning from the instructor. The following interview excerpt illustrates how one student felt about working in a group-centered environment in which the instructor did not lecture.

> I think it is good for me because I have never been good in math, and [it is] something I have always hated. And so it's been good for me, because it is giving us someone to turn to besides the professor, for understanding the material. So my whole group has helped me out, kind of, instead of my professor. And the problem with only having one professor, too, is that professor isn't always available to help me when I want it. So, if I know the people in the class, I can turn to, it helps a lot.

Rather than relying on an authoritative person or text for the mathematical knowledge, students constructed their own knowledge through group collaboration. In the following interview excerpts, students describe the process of group learning.

> And each of us knows a little bit about something, like there was one thing the other day that I was just kind of like, "What is this? I can't even come close to knowing how to solve this!" But then somebody else would know. And then there was another day where I knew right away how to solve something, and I was like, "Oh, hey! We can try this." And so it was really neat working together because somebody always had something to add. And rarely did we come across something where we're all like, "Huh?" Or we don't know anything. I mean somebody always knows a little bit of something. And even if they don't know the whole way to get to the end of solving the problem, even having that

little bit of knowledge. Somebody else can take their little bit of knowledge and build off that, so that's what's fun about the class.

* * * * * * * *

I liked [group work] because everyone has their own little way of thinking about things. When you look at a problem and you get stuck, it is really hard to direct your mind in another way, but when you have four people sitting around together and they all have their own minds thinking in their own ways, you can piece things together and learn from each other.

These students are describing a process in which they work together both in small groups to solve the problems. By constructing the knowledge rather than receiving it from the instructor or a text, the students come to understand and know the material more deeply and are more engaged in the discovery of the answer. In addition, through working together the students gained confidence by realizing that they could come to a mathematical solution on their own.

3. Shift in the relationship to knowledge

The Math 130 learning environment centered on getting students to create their own mathematical knowledge, rather than receive it from an authoritative external source. In so saying, it is important to stress that this environment was carefully constructed by the instructors. By shifting the source of knowledge to the students, the instructor creates a shift in who "authors," or has authority about that knowledge. Gamson discusses how the process of "collaborative learning leads to changes in authority relations between students and teacher and between students and knowledge" (See Gamson[1] and Bruffee[2]). Many of the students we interviewed expressed a degree of ownership of the material they had learned in the class.

My experience with 130 and 131, because I have already taken 131, is that they are really teaching about math, in a way that is not just writing down numbers and solving in the way that you traditionally think of math. And I think that is really interesting, and I think that with us, like other classmates will sit there and try to figure it out and try to figure and it will be really challenging. But once you do, it is like, "Oh my gosh, I just got this!"

Many of the students we interviewed commented that they were more engaged in learning the material in this class and contrasted this with their experience in more traditional classes. In the following interview excerpt a student described the level of his/her involvement doing the mathematics.

The class always went fast. I never dreaded going to the class; I wasn't like, "Oh, I'm going to

[1]Gamson, Zelda F. (1994). "Collaborative Learning Comes of Age." *Change, The Magazine of Higher Learning*, September/October 1994, 44-49.

[2]Bruffee, K.A. (1994). *Collaborative Learning: Higher Education, Interdependence, and the Authority of Knowledge*. Baltimore, MD: Johns-Hopkins University Press.

be so bored today!" Usually I was interested in what I was doing the whole time I was there, and I would leave class maybe thinking about what we had been doing and wanting to go and work on it some more, because it kind of carried over, especially if we came close to the solution at the end of class.

This student described a high level of engagement with the mathematical knowledge. For some students this engagement resulted from experiencing themselves as active participants in the learning process (see Tab 5, Section A.4 for more information on engagement in the learning process). This was transformative for some--They were not simply "taking a math course." They were "doing math." In the traditional lecture format where the instructor delivers knowledge, s/he may "stand between" the student and the knowledge. In contrast, where the instructor facilitates a collaborative, active learning environment, the students are directly engaged with the knowledge.

What I'm saying is that you come in and you don't really (pause) necessarily understand exactly what's going on. You're given these problems and you have to write them up and you understand why, because you have to do this as a teacher. But even if [the instructor] told you, I don't know if you understand why it is that you attack things the way you do, why you're sitting there trying to do this, and [s/he's] not doing much to help you. I think even if [s/he] would have told me, "I'm doing this so you learn how to solve problems on your own." It really hadn't sunk in for me. The second half and I'm really not sure why, it really just started to sink in to me. And it came home with that question on the final, you know, "I've never seen this before." But I've been given tools in this class to use on these things, and I used them, and I solved it. So I mean it really said to me that this is what the class was after, this is how [the instructor] taught it to get there, and I just appreciate that a lot more.

4. Experiencing the shift results in increased confidence

Through the process of working in groups and struggling to come up with solutions without receiving a method from the instructor, many of the students gained confidence in their abilities and their understanding about mathematics.

[My understanding of math is] very different. Very, very much different. I mean, before I would have never have talked about understanding foundations of learning math. [The instructor] taught me the first time, or Math 120 was the first class that really taught me or showed me that there is a lot of stuff that you just know internally, that you can learn by yourself without having something set in front of you. I guess I was so much more comfortable with having something set down in front of me. And now with my math, I'm very confident teaching mathematics for young children. It really applies back to my Math 120 class, the way that we learn in that class and reasons behind learning that way.

Many students who perceived themselves as poor in math benefited from the realization that they could solve problems without direct instruction from an external source. The following interview excerpts two students illustrate the impact Math 130 had on their self-confidence in relation to math.

R: When we all agree on something, it makes you feel really good because you figured it out on your own without a professor's help, and not every group gets it. Sometimes our group doesn't get it, and we're like, "Why didn't we think of that. It's so easy!" I think this whole aspect is just very -- I'm glad I took it. It has boosted my confidence.

I: Confidence in general, or confidence in math?

R: Confidence in math. I don't feel as inadequate as I used to. I can do this kind of math, and I can do it without feeling bogged down or anything.

* * * * * * * *

It was really experiential. It was kind of hands-on, figuring things out. No one right way to solve-- problem solving skills was basically what the class was. Different ways to look at things and work through things. It wasn't a real rigid kind of, "I am going to do this and you are going to do it that way." So it was much more flexible and individualized. And initially just as terrifying as any other math class for me because I do have a math phobia. But after about a third of the way through, I just felt much more comfortable and thought more in terms of "How can I figure this out? How is this easiest for me to do?" And the fact is that I wasn't doing it the way anyone else was doing it, but I was coming up with reasonable responses.

Many students felt that working in groups and explaining mathematical concepts to their peers increased their understanding of, and self-confidence in, mathematics.

I feel a lot more confident. Through the steps I feel more confident in my ability to really understand what I'm doing. Because the write-ups and the format of the class and that talking in the groups has forced me to explain to other people why I understand a problem. And then I know that I understand it. So then you know it proves to myself that I can do that.

All of these students are describing how, through the learning environment created in this class, they come to develop confidence in their own learning process. For some this process was transformative as they turned their new confidence towards understanding mathematical material that had been previously inaccessible. Thus, the shift from relying on externally affirmed solutions to constructing the knowledge through their own process, resulted in an increased sense of both confidence and ownership of the knowledge for many students.

B. Issues Involved in Adapting to the Math 130 Culture of Learning

Certain difficulties arose as students adjusted to the Math 130 teaching and learning format. This new course culture required a significant amount of adaptation on the part of students. We found that some students adapted rather quickly whereas others never felt comfortable. We came to understand that many of the complaints and problems that the students repeatedly discussed were related to difficulties in adapting to a new course culture. In the following section we discuss the most salient adaptation issues that arose out of our interviews with students in Math 130.

1. Adapting to the "Instructor as Guide" teaching process

The Math 130 culture of learning required a degree of adaptation by the students. Some students welcomed this shift and were very comfortable with the process of discovering knowledge themselves. Other students had a very difficult time and felt like they were wandering without any direction at all. The following interview excerpt raises typical problems that some students encountered in this new course format.

> I remember in the first group that I was in, we didn't know how to do a problem at all- no clue at all. I can't remember what it was on. We asked [the instructor], "Can you help us a little on our --," and [the instructor] said, "No, I can't give you any help, any answers, any help." Well, then what is the purpose? I mean, it just seems like, "Great, you can work in a group, but if you don't know how to do it, it's not just going to magically come." You know, if you have no background information in there, if you have never had any of what you're doing, it's not just going to all of a sudden appear in your mind and you're going to know how to do it. So, in that way I guess I'm really disappointed and leave frustrated a lot.

For some students not having the instructor provide direct and concrete assistance on how to solve the problems was a large source of frustration. Many of the students who experienced these frustrations mentioned the problem of not having enough background in mathematics to solve the problems. These students were very resistant to the new role of their instructor. They felt that the instructor should provide them with the background information and, if necessary, the method by to solve the problems. Some students did not understand the role of their instructor in this class and questioned the effectiveness of having someone just ask questions.

> R1: That class drives me crazy.
>
> I: How?
>
> R1: It frustrates me. The whole class just frustrates me. How we go in there and we'll work on the problem in a group, and--that doesn't bug me. But then we'll start talking

about it, and we'll never get the answer to the problem. [The instructor] will never tell us the answer to the problem. It drives us crazy.

R2: Yeah, it's frustrating!

R1: And then [the instructor] gets mad - it seems like [s/he] gets mad and frustrated if we do find the answer, and -- (sighs)

I: What do you mean?

R1: [The instructor] seems like [s/he] always tries to do things to keep us from not getting the answer. Like, we'll have the answer there in front of us, and [s/he'll] be, "Oh, what about this?" or "What about this?" which maybe [s/he's] supposed to be doing that to try to get us to think about it, but it gets frustrating. You get so sick of it.

Instructor support an important component for student adaptation

Many of the students who expressed frustration were happy with the group work overall, but wanted more of an indication when they were on the right track. This was a factor for students of varying mathematical backgrounds. Some students who had previously done well at mathematics were experiencing difficulty and felt that they were doing worse than they had done in other more traditional math classes. Other students who had never felt particularly strong at math felt unable to assess how well they were doing in the class.

I: What do you think that [the instructor] could do differently to make the course better for you?

R: Let us know if we're understanding it the right way, because if we're not understanding it the right way, it's not going to do us any good.

The process of learning to trust their own answers took a long time for students who had never been in a situation like Math 130 and not all students ended up feeling confident about their mathematical ability. This raises the issue of the multiple roles of the instructor in this new course culture. In addition to the role of the instructor as a guide in the problem solving process, an essential part of their role is to assist students in adapting to this new method of teaching and learning. This role is perhaps less obvious but certainly should not be minimized in its potential impact on assisting students in the process of making the shift to this new culture of learning.

R1: It gets really frustrating. You get no support in that class, and you're just like, "I'm failing, I'm failing, I don't understand anything," and it's just no support. It's just well,...

I: Is there anything that [the instructor] could do differently that would help you?

R2: Yeah, [the instructor] could be more supportive. Like if you're doing something and

maybe along the right track, trying to figure something out, say, "That's a good way to try," or, "You're doing a good job." But [s/he] doesn't. [The instructor] will just sit there and stare at you, and walk away. [S/he'll] just stare at you and walk away!

This focus group discussion illustrates the importance of the instructor providing some system of support for the students. Adapting to this new method of teaching and learning can be unsettling for students and the instructor can play an important role in preparing students for the potential difficulties they may face as well as assisting them in understanding the adaptation process.

Adaptation required a balance in the amount of help given

This "culture of learning" will not thrive if teachers give too <u>much</u> help or if they give too <u>little</u>. The interviews and surveys indicated that when instructors left the class hanging too much, without providing support and feedback about whether the students were on the right track, the students felt considerably worse about the course and their own abilities at mathematics. It appears, moreover, that the instructor will discourage students from accepting and understanding this new learning environment by providing too little help and support in adapting to the new method. This is particularly true if the instructor, intending to push students to learn on their own, ends up leaving them with no framework by which to assess themselves. The following excerpt illustrates this point.

I: And you said, sometimes that it's necessary with certain concepts to have guided instructions. What do you mean by guided instructions? What do you mean by that in the class?

R: Well, like just an explanation of the concept. You know, how does it work in the greater scheme. With something new, you almost always need some sort of a definition of what it is and how it works before you can use it. So that's what I mean by guided instructions. I think that the frustration level in the class might decrease if the people that aren't as comfortable with this kind of learning, if [the instructor] would give out an example of the correct answer and the solution to the right answer. That might be a good way to get people to see where [s/he's] going, what [s/he] wants, and it also, might help spark people's learning a little bit.

I: So you mean to have an example, particularly with a new concept to show an example of it. For example, on the board being worked out.

R: Not even so much that actually. You know, for example, maybe the first couple write ups that [the instructor] would have the students do. After [s/he] maybe hands it back with the points, also hand back what [s/he] would consider to be the solution, and an example or an explanation of the solution...I think that would release some of the frustration. It would also sort of clear things up, and make it easier. I think the way, especially at the beginning of class, I met some people, it's not just me that thought that, "[The instructor's] not giving us the answers," you know. And --

> I: And people were frustrated....it was frustrating that [the instructor] wasn't giving them the answers.
>
> R: Yeah, it was like, "I don't know if I'm right or if I'm wrong or if I am doing this right." And I kind of feel like, if it's not wrong, then I did it right. But I think for the people who didn't do well, they didn't know necessarily why or what to do about it.

In contrast, by giving too much help, the instructor reduces or even eliminates students' motivation for solving the problems. In the following excerpt, a student commented that receiving too much help removes the excitement of, and motivation for, learning.

> Yeah, [the instructor is] great. If you have questions [s/he'll] help you out. I think if [s/he] helped us more, like gave us -- because usually there's one thing you're not thinking about and after you get a problem started then it's so easy to solve and you're like, "Oh, why didn't I see this in the first place." So, sometimes if [the instructor] helps you out too much on a problem, [s/he'll] just give it away and then it was kind of not worth it. But I think [s/he] walks the line pretty good between telling us nothing and too much and I think [s/he] does a good job.

Our analysis indicated that in some sections, students were provided more help than in others. In our interviews with students, we found that students in some sections were able to rely on the instructor to supply for them the tools to solve the problems. Our interview data suggests that in most cases, when students were given all of the background information and examples to solve the problems, rather than being left to discover these tools independently, they did not find the problems as challenging. As a result, they often did not acquire the same depth of understanding of the process of problem solving as those that worked more independently.

The instructor's ability to ask questions that provoke thought, to guide the students instead of letting them work completely alone, and also to reassure the students that their efforts are worthwhile has a significant effect on the students. That the right balance between too much and not enough guidance can and was achieved, at least in some classes and for some students, is conveyed by this student quote.

> I would think one [role of the instructor is] to provide a little background when there was less knowledge, to help lead us in the right direction. I know there were times we were working in groups, and we are just so far off course, and [the instructor] would come and [s/he] would just ask a question, "What about blah, blah, blah?" or, "Have you thought about this approach?" It kind of keeps us on track, I think. I think sometimes it was slightly frustrating. [The instructor] would stand back and be the impartial observer, or play the devil's advocate, like, "What about blah, blah, blah?" when we're all like, "No, we're right!" I think that also when [the instructor] would do things like that, you kind of notice that towards the end of the class people were -- when [the instructor] would try and lead us in the wrong direction, people would be like, "No, you're going the wrong way," and I think people were a lot more sure of themselves by the end of the class.

I: Do you mean by the end of the semester?

R: Right, by the end of the semester. I think for me it helped boost my confidence in my math, definitely, cause I was finding the solutions, and when I wasn't sure, I wasn't frustrated to the point that I couldn't do anything about it. I was given enough support that I could use whatever material [s/he] gave me, so I think it definitely helped to boost my confidence....I was the one who was working through, or me and my group were the ones who were working through, these problems, and somebody wasn't just giving me information. There's still confidence building in being able to take the information given to you and use it across the board, but I think that because I was just sitting down and doing it by myself and being successful at it, I think that was an important part of it, that I was able to be successful. I think if I was always frustrated and I was never given enough information, if [the instructor] didn't provide enough clues, I think I wouldn't have felt so confident....I was finding the solution, and I wasn't frustrated. I think that's an important part of it too. Not only was I finding the solutions, but when I did run across frustration, I was able to overcome it, I was given enough tools, either through myself or from my classmates or from someone else, that I could overcome the frustration. I think that in case if I couldn't have, I wouldn't have felt this confident.

I: What do you think helped for you to not get so frustrated? What were some of the tools you used, whether they be - I don't know what you're thinking of that helped with that frustration when you encountered it.

R: I think classmates helped a lot. Classmates were more likely to say okay, this is how you do it, do you understand? That definitely helped. I think that [the instructor] did provide enough support for me, if a classmate couldn't explain it to me, if I couldn't figure it out sitting at home, then I could go to [the instructor] and say, "This is not clear," and [s/he] would provide enough information.

This student is describing how this balance between giving too much and too little help affected his/her learning process. When students were provided with the right amount of support that enabled them to adapt to this process, it resulted in increased confidence in their problem solving skills and a more successful class experience overall.

2. Adapting to working in groups

In Tab 5, section A.5 ("Group composition") we presented some of the concerns that students discussed about group work. As we noted, in these discussions the students often attributed problems to the composition of the groups. Yet interview data also indicates that working collaboratively is a learned process and that students need guidance as they adapt to this new style of learning. Students generally were accustomed to working individually and relying on the instructor for problem solving methods and answers and therefore were uncertain about how to work collaboratively.

> Here we sit down the first day. Okay, draw a card. Okay, you're number four. You are in this group. We have never met each other. We are sitting down and suddenly -- I don't think you can start a cooperative experience without knowing people, trusting the people. You know, I think that is part of it. And if you just plunk people down they are probably going to work just the way they are used to working, which is independently.

When confronted with the different learning styles and mathematical background levels of group members, students tended to work individually or align themselves with those of similar ability. The following quote appeared also in Tab 5, Section A.5 ("Group Composition"); it is repeated in this section because it best represents the experiences of some students.

> We switched groups and so I have been with the people in that group, and two of the people were not, they were very weak in the math department, and it was kind of frustrating that they would never speak out when we were throwing out ideas. They were kind of just silent and into themselves and it sort of even broke off between, there was just me and this other girl who were contributing, and they just kind of broke off by themselves. I thought that the point of the group work was to be contributing, so even if you didn't have something to say, you could say, look at that, or what about this. So they weren't really saying anything, so it was kind of hard to facilitate a four-person group. We would kind of split off into two person groups. Then [the instructor] would come over and say, "Why don't you explain it to the rest of your group," and we usually were working on completely different things and when we showed it to them, they sometimes understood, and they sometimes didn't understand.

Interview data suggests that one part of student adaptation to working in groups was learning to work within a framework in which there are individuals with multiple levels of mathematical experience and varying degrees of confidence. For some students, these factors compounded the difficulty in adapting to group work and at times even prevented them from viewing their group as a resource. In the following two interview excerpts, students discussed this issue.

> Being in a group it depends so much on the people that you have in your group and their background in math. If you had, let's say, a group that didn't have a lot of knowledge about math, then it's hard for that group, and that can sometimes bring down a group. It depends so much on the individual, whereas the older way of teaching, the traditional way of teaching, you had a teacher. You had math books that you could turn to, and now you don't. You don't really have resources here.
>
> * * * * * * * *
>
> I just don't have a firm enough of basis to do well in math and I don't think -- like working in these groups -- It was almost like the blind leading the blind.

These students expressed the view that groups did not always provide the consistent knowledge base that they perceived as present in the traditional method. In Math 130, without lecture and text, students who were unable to view their group as a resource experienced alienation from the learning process. The following interview excerpt illustrates this experience.

> And our groups, first I worked with a group -- I was just working with them for that week -- and they just worked on it on their own, and I just sat there like a jerk for twenty minutes. And then my other group was doing the same thing. I have a friend in the group, and she felt the same way. They would just sit there and start working on it.

This student continued by commenting that the instructor played an important role in the facilitation of a collaborative learning environment.

> And I was confused whether the goal was process or product. So I said that to [the instructor], "What is the deal?" I know a lot of professor-types don't understand about cooperative learning. They think group learning - you throw people together and they are going to work together. Well, we have all worked individually our whole lives. We don't know how to work as a group. So [the instructor] really stressed that in class--which was great...Now [the instructor] is doing something because [s/he] is very crafty and stuff. And [the instructor] is coming to each group and sitting down. Last week [the instructor] sat down with every group for ten minutes. I know what [s/he] was doing. [The instructor] didn't have to say anything. I know [s/he] is watching to make sure our group is working....Maybe it takes time, but people just don't naturally work together.

In summary, students were accustomed to an individual learning process and needed assistance in making the transition to working collaboratively. Many students we interviewed explained that the instructor needed to play a role in facilitating this adaptation in order for the students to work collaboratively.

From the Instructors' Point of View: Issues Involved in Teaching Math 130

In this section, we will discuss the instructors' views of the goals of Math 130, their role in the course, their teaching strategies in achieving the goals, and also their attitudes on the curriculum. The Math 130 instructors were interviewed the semester in which they taught the course.

It is important to note that teaching Math 130 differed from teaching most other math courses at the university. The instructors were asked by a committee, rather than assigned to teach Math 130. In addition, an attempt was made by Professor Pemantle to inform these instructors about the philosophy and methods underlying Math 130 by requesting that they attend meetings with current instructors as well as requesting that prospective instructors observe the course a few times before teaching it. Therefore, in Math 130, the instructors' goals and teaching methods were informed by the underlying philosophy of the Math 130 method.

A. Goals and Philosophy of Math 130

In part, due to Professor Pemantle's efforts to convey the philosophy underlying Math 130, there was general agreement among the instructors about the goals of the course. However, it should be noted that while the instructors shared these goals, they differed in which goals they considered most important and also in their strategies in achieving these goals. In this section, we will discuss each of these goals.

1. Changing students' relationship to math from fear to confidence

The instructors noted that many of the students entering Math 130 have had little math experience and also had some anxiety toward mathematics. All of the instructors stated, either explicitly or implicitly, that one goal of the course was to change the students' attitude from one of fear to confidence in their ability.

> [The students] are changing their attitude from what it starts out with, which is fear, resentment -- resentment that they have to do this stupid stuff, fear because they think they're not good at it -- and lack of interest. So its all these things that you start out with, and you are trying to change it. I don't want them to all come out glowing about math -- I mean it would be nice, but not realistic. But in particular, you want to get rid of some of the fear. You want to make them feel this is something they can do if they want. They don't have to like it, but they can do it. You want to get rid of the attitude that there's some kind of mystique about mathematics that's pretty negative that they have that mathematics is this difficult, cold, external sort of subject. And change that into a feeling that mathematics, and individual mathematics problems, are things that one can think about

without being told exactly what to do. That one can reason with, relate to and ultimately come up with good ideas for, whether or not you learn how to solve the problem completely.

The instructors explained that through the problem solving approach that students could move beyond fear of math to a deeper understanding of the process of "doing" mathematics. Several instructors expressed that getting rid of this fear was important in order for the students to become effective teachers.

I think I want them to lose some of their fear of math, that's important, and lose some of their antagonism toward it, toward problem solving, because I don't think that's a good thing to bring to elementary school teaching.

* * * * * * * *

And I'm imagining what sort of attitude they are going to convey to the children and you want them to convey the attitude that you get a math problem, you try, you think about it, play with it a little bit.

2. Facilitating student understanding of mathematics: getting beyond the facts

The instructors noted that, in addition to having anxiety about mathematics, many of the students didn't understand the nature of math and often considered it to be a series of formulas that are simply memorized for an exam. Thus, the instructors expressed that a second goal for the course was to broaden this perception.

I believe that many people who take mathematics don't know what it is about. They really don't know what the nature of the subject is, like why it might be beautiful or interesting, or what is creative about it, or why might you even want to teach it? Other than to balance your checkbook or to figure out how much money you owe on your rent. Really that is not very interesting, right. This is why I was open to Math 130, the idea of it. I was becoming very convinced that a lot of my students would just have no clue as to what the subject was about. They felt that it was a bunch of formulas that you would memorize so you would use them appropriately on the exam. You might use the problems to solve, but math for math's sake was missing. Really understanding what the subject was about as far as the how you need to build a convincing argument for something in math. How it all connects together, the connections that are there. The logic that is there. It just wasn't seen by students, so one of my main goals for the course and my philosophy for the course it is to convey what I could say was mathematics. And the reason that I would want to do this for teachers, is because I think that it is very important that if someone is going to teach a subject, to know what the subject is. Not just be able to add and subtract and multiply, but just have a sense of what mathematics is about. And if you have that sense, when you are a teacher, you get appreciation for the subject. If you really have it, you can't help but convey it to your students.

For this instructor, when students realize that mathematics allows for creativity, they are able to explore ideas within the framework of the logic and become mathematical thinkers. All of the instructors commented either explicitly or implicitly that students who became mathematical thinkers developed a skill that was directly transferrable to other areas outside mathematics, especially teaching.

It was noted by some faculty members that students themselves were aware of their need to acquire a better understanding of mathematics particularly because they are prospective teachers.

> So the 130 students are prospective teachers. Some of them are turned off to mathematics, although not as many as we would expect. Some of them are quite happy with it. They are very unsophisticated with their mathematics and thinking skills. But they do agree, essentially, that it is something they ought to know, and that makes all the difference. So the clientele is mostly willing and I also happen to think that is one of the most important places for beginning any project of improving math overall -- to begin with the elementary school teachers because they have their hands on the kids for years and years.
>
> * * * * * * * *
>
> R: Yeah. I mean, they were aware that there was this changing coming up, right - that education was going to be changed, and that now in schools you have to teach the reasoning and not memorizing math. Many of them have told me in class that they were actually afraid because they were not ready.
>
> I: They were not ready for what?
>
> R: For that kind of education -
>
> I: To teach that way?
>
> R: Right, because they always memorized everything, and they didn't have to understand things. When we were teaching, for example, the rules of arithmetic - how to multiply and how to add. So we were teaching that, and we were teaching, "Why do you do that? Why don't you just do something different?", right? They were actually very surprised. They never thought about why would you do certain things when you multiply and when you divide. It's kind of funny. You are going to teach that idea, and you don't know why you are doing what you're doing?

3. Facilitating the development of student problem solving skills

All instructors of Math 130 wanted the students to be able to solve problems by thinking through mathematical ideas logically. Several explained that in order for this to occur the students needed to develop their problem solving skills. One instructor articulated these skills.

> Namely, teach them actual cognitive skills, which are problem solving strategies--how to remember what problem you are working on, how to use trial and error, how to reduce the

problem by analogy. Look at a book on mathematical problem solving and you'll see all of these different skills.

Several instructors observed that when students developed problem solving skills, they would acquire the ability to think through a problem without depending on an external authority. One instructor felt that the development of these skills and the concomitant independence was the most important goal of the course.

R: Oh, I would be really happy if at the end everybody would just sit and think about [the problems].

I: Instead of what?

R: Instead of just getting stuck and saying, "I never saw this," which is the usual problem. (laughs)

I: So that would be something you would want them to acquire?

R: I would be happy actually if they acquired just that!

I: So that would be enough!

R: In a constructive way, of course, if you think about it. But if they are able to see it in any problem of any kind, to actually see, then say something constructive about it, that would be good. Even for them, I mean, for their own self-esteem.

The instructors explained that the knowledge of the problem-solving skills and the confidence that is concomitantly gained would enable students to tackle problems that they haven't encountered previously. These cognitive skills are not limited to the area of mathematics and are particularly transferrable in all areas of teaching.

I think they're going to have to teach a certain amount of material, and part of the problem has been that I think teachers, if they run across material that isn't presented just the way they know it, or material that they really understand, it isn't very easy for them to teach it. This practice with having to work out these problems with each other hopefully will put them in such a position that if they're given a book, where certain things are unfamiliar, they'll be able to sit down and figure it out. And perhaps also, since they've learned to deal with other people, figure it out with other people. Maybe there are three teachers in some school that are having to deal with a certain new book at a certain grade level. Hopefully with this kind of experience behind them, those three teachers will get together and sit down and work it out.

4. Increasing students' knowledge of mathematical concepts

The instructors expressed that a fourth goal of the course was for the students to learn some mathematical facts. All instructors explained that through the process of problem solving, students developed a greater understanding of the facts that they discovered. However, there were differences among the instructors when deciding the amount and type of mathematical facts the students should learn. Some instructors believed that the students should learn a specific body of mathematical knowledge. Others did not emphasize the learning of a particular set of facts.

Learning a specific body of mathematical knowledge

Some instructors expressed that a goal of Math 130 was for the students to learn a specific body of mathematical knowledge.

> I do feel that there should be a body of knowledge on the whole that you're transmitting. It shouldn't just be a bunch of techniques. I don't think it should be a class in problem solving itself. Or, well, if it is, then that's one thing. But if it's a class in mathematics itself, then there needs to be a certain amount [of knowledge transmitted].

These instructors observed that the inquiry-based, problem solving approach to mathematics in Math 130 does not guarantee that the students will acquire a knowledge of specific mathematical content areas. Thus, in order to ensure that the students learn the material, these instructors commented that they should, if necessary, channel the students' inquiry in the problem solving process.

> But the way I look at it, we have a finite amount of time left. And [if there is] too much going in the wrong directions, [then] they're not going to learn as much.

Little emphasis on learning specific mathematical facts

Other instructors did not feel that an emphasis should be placed on learning a specific body of facts; rather, their primary goal was to develop in the students a problem solving approach to mathematics. The following instructor discussed his/her experiences in using a non-guided inquiry-based approach to having the students discover base two.

> And I knew what was going to happen because I had done it two times before. And that was fine with me that they wouldn't come up with base two. And at the end, [a Math 130 observer's] suggestion was how to have made it so they would come up with base two in a very direct way, like in a pretty way. And my comment was, if my whole goal was for them to know base two, maybe [s/he] is right, maybe not, maybe so. That was my comment, but my objective was the process of how you might go about thinking about mathematics and I was looking for all sorts of other things than base two.

These instructors felt that the skills students acquired when they developed a problem solving

approach to mathematics through an inquiry-based process would prepare them to not only learn mathematical facts in the future, but also the reasoning that underlies that knowledge.

For more information on how these approaches to learning mathematical facts influenced the instructors teaching strategies, please refer to Tab 7, Section C.2 ("Transferring responsibility for student learning").

5. Increasing the students' ability to explain mathematics

All of the instructors, directly or indirectly, alluded that a fifth goal was to teach Math 130 students to speak mathematically. To develop this skill, the students were assigned to work in groups, giving them the opportunity to explain problems to one another. They also were given writing assignments in which they needed to explain their procedure in finding a solution. One instructor articulated why the goal of learning to speak about mathematics was particularly important for the students of Math 130.

> And last of all, for teachers, what is really important is that they learn to speak about mathematics. That they learn to say things that a mathematician would cringe at. And in fact even become eloquent about speaking about mathematics. And mathematics is it's own sub-culture, it has language and it has vocabulary and it has standards of finding validity. You need to be exposed to a little bit of that if you want to understand. If you are going to talk about math about math with kids, then some experience when talking about mathematics is appropriate. So that would be the third, big part of why I do what I do for teachers.

We should note that the goals chosen by the instructors were interrelated. For example, the instructors described how exposing students to the process of problem solving would increase their confidence in, and understanding of, mathematics while decreasing their fear of math. In addition, the goals emphasized by the instructors were, in part, shaped by the students being prospective teachers.

B. Instructor Role: Creating an Environment for Student-Centered Learning

Through observations of the course, interviews with the students and instructors, we came to realize that although the instructor played a less active role in Math 130, s/he was critical in facilitating an environment in which student-generated learning can thrive. In addition to integrating the different methods of learning in the class, such as group work, class discussion, and the write-ups, the instructors facilitated the students' transition into this new way of learning, guided them to achieving the goals, and prepared and executed the curriculum. Our interviews and observations of the course indicate that while all instructors incorporated these aspects into their role, each did so to a varying degree. In this section we will discuss the multifaceted role of the instructor in the student-centered environment of Math 130.

1. Providing support

While all of the instructors provided some degree of support for students during their transition

into the new teaching and learning environment in Math 130, the amount and type varied. In this section we will discuss the different types and amounts of support the instructors provided for the students.

Some instructors considered providing support a primary role in Math 130, commenting that all students would need some support during this transition. In the following interview excerpt, one instructor demonstrated how s/he addressed the entire class when providing support.

> And, I also tell them, "You really do have all the support from me you could ever want, really, because I'm here for appointments. I'm here for office hours. I'm really here for you. Your group's here for you. Don't feel like you're alone. So, it's not like you're alone, and here I'm expecting you to solve this problem. And then, I haven't told you anything about it and I'm going to grade you on it, right? You have support in place, and so, come see me if your worried about a problem, and I sympathize, I know it's hard. You know, you're doing fine, I think you had a great insight here."

While all instructors provided support by reacting to students' frustrations and anxiety with words of encouragement and/or explanation, many instructors provided support for the students only as the need arose and did not consider it a fundamental aspect of their role. For many instructors, the support given to a student or group of students depended on the student and his/her level of frustration.

> R: I responded at first with more feedback than some people might have because I felt like they needed encouragement. I felt like their tolerance for frustration wasn't very good at first. So I did give them positive feedback -- "Yeah, I think you're on the right track." It depends.
>
> I: What does it depend on?
>
> R: It depends on whether I think -- this is a judgment call on my part -- they will work on it if I don't give them the answer anyway, or whether I think they're so frustrated they need encouragement.

One area in which many instructors felt that students needed reassurance was in assessment. Because of the nature of the course, many students lacked a coherent sense of what they had learned. One instructor commented how s/he provided support for students who had a difficult time preparing for an exam.

> R: There is no list of subjects to start with, because it's just activities that we go through. They are not sure what they learn because after an activity they have learned a lot, but they don't know what because there is not a list of things that they have written down. I actually gave them a list of things. I said, "This is what you have learned. You have learned this and this and this and this."
>
> I: Why did you give them that?

R: ...So they would feel better.

Having neither lecture notes nor a textbook, students often felt unsure of what they had actually learned. This instructor provided support by reaffirming that learning had occurred through the group work and class discussions.

In summary, each instructor decided for him/herself the amount and type of support to provide. For some, providing support was central to their role and assumed that all students needed support in their transition; others provided support in reaction to students' needs and frustrations.

2. Facilitating group discussions

The instructors explained that group work, when used effectively, played an important role in facilitating the student learning. Our observations of the course indicated many instructors monitored the groups to ensure that they were on task, and also facilitated student learning in the groups by asking questions and guiding students who were stuck into a more fruitful discussion.

However, our observations of the course and interviews with the instructors also indicated that many of the instructors did not have as a primary focus the facilitating the students' adaptation into working collaboratively. In fact, only one instructor specifically commented on the methods s/he used when dealing with group dynamics.

R: Another thing you do is help with the group dynamics. If one student wants to dominate then you do things like, you come into a group and you say, "You, Nancy, tell me what your group is doing." You pick an arbitrary person, but you avoid picking the person who did clearly the bulk of the work, if there is one....In 130, it happens a smaller proportion of the time.

3. Bringing out ideas in the class discussion

The instructors considered the class discussion to be a forum which allowed students to experience the process of problem solving either by witnessing the different methods other students employed in working toward the answer, or perhaps by finding a solution as a class. Thus, one role of the instructor in the class discussion was to draw out the key ideas, thereby guiding students toward a solution. One instructor commented on his/her strategy in getting students to discuss ideas relevant to the discussion.

I sit, I make sure that the conversation includes everyone. That is kind of my job, and then everything relevant that people know, gets said. So if I know that someone is holding back for example, I will say, so "Liz, didn't you find something like this. What was it that you found?" Or if two people start talking about something that is really interesting, I might say, "Nikki, can you tell us all what you are talking about, so that I can hear that, so let me get this straight, so Nikki you are saying this," you put it out there, like what is out there.

Another instructor expanded on this strategy by bringing out the ideas in an order which reflected the problem solving process.

> R: I try to call on groups which have rudimentary findings first to get those fully aired before. Once you see a group that has a pretty sophisticated approach it is hard to go back.
>
> I: So you try and bring out, you try and know where each group is.
>
> R: Yeah, you have to know or else.
>
> I: Especially a lot of observation you yourself do all the while.
>
> R: Yeah, and that is why a lot of us let it occupy the [remainder] of the first part of the class. You notice what kinds of things might come up for discussion. For example, some groups really have a completely different idea of what the problem is asking. That is a good thing. You don't want to just sweep that under the carpet. That is the best thing to bring out in the discussion. Sometimes one group has a method that will work but it is too sophisticated for them so they can't make it work. So you leave it there and you say, "That's interesting." And you go on and discuss other stuff. Sometimes that rings a few bells, and they come back and say, "That's what we were really trying to do."

By reenacting the process that the students went through in their discovery, the instructor perceived him/herself as allowing students to view the process of problem-solving -- to see ideas being formed and refined into the end product.

C. Teaching Strategies

While all of the instructors had similar goals for Math 130, many prioritized the goals in different ways and also used different strategies to achieve the goals. In this section we will discuss the varying strategies the instructors used in the course.

1. Providing a framework for the course: orienting and motivating students

Some instructors anticipated resistance to the format of Math 130 and attempted to reduce this by providing the incentive to the students that this new teaching and learning environment would benefit them as learners and as future teachers. One instructor used the term "enrolling" to describe the process of getting students involved in and excited about their role in the learning process in the course.

> I: What do you mean by [getting the students enrolled]?
>
> R: I mean, on the bandwagon with me, like they're on my side, like they're, "Yeah, this is the way to go." Get them to where they're the ones who are creating the course along with you.

In order to get students excited about the course, some instructors explained that the students needed to understand the benefits of this new teaching and learning environment. The following interview excerpt illustrates how one instructor attempted to "enroll" the students.

> I said, "It will ensure your participation in the class much more than if you're just listening to me talk, and that it will help you deal with that feeling when you see a problem that, 'Oh, I can't solve that,' and just giving up. You will get over that because you will learn to persevere and you will solve the problem." . . . I said it would be a great help that way, so I said it from those two things -- "It will be much more engaging, and it will help you overcome your fear of problem solving."

Some instructors "enrolled" the students by explaining to them that they were the crucial element to the Math 130 learning environment. The following interview excerpt demonstrates how one instructor encouraged the students to take an active role in the class.

> I give them speeches every once in a while, about, "Look, this mathematics that we're doing here. These problems that were doing, we're creating all this in this classroom right now. And I'm not doing it. You are doing it. We're doing it together. You're really important, and your working and thinking on these problems is what makes the course work or not work. You're responsible, and I need you. I need you to help me." And I'll just say it like that sometimes, not, "Okay, you guys, c'mon ask a question." But, I'll say, "C'mon you guys, help me here. What's going on? What needs to be said?" And, they do.

By giving students the framework and motivation behind the course, these instructors were orienting students to the new teaching and learning environment. These instructors felt that students who were "enrolled" were more likely to accept the new format of the course, thereby changing their attitudes toward their relationship in the learning process. Rather than expecting to be given the methods to solve the problems, students might begin the struggle to become problem solvers.

2. Transferring responsibility for student learning

All of the instructors consciously worked to shift responsibility for learning to the students in order to facilitate the students' transition into independent problem solvers (for an in depth discussion of this shift, see Tab 6, Section A, "Shift from instructor-centered to student-centered culture of learning"). One instructor discussed his/her role in this shifting of responsibility for learning to the student.

> R: On another level, the role of the instructor is to try to make this work. One of the things the instructor has to do is to try to get them working in groups and not depending as much on authority. When you start the course, there are always some students who will call you over and say, "I don't remember how to do this kind of problem. Can you explain it to me?"
>
> I: And then what do you do?
>
> R: And then you say, "Well, what do you have to do to figure this out?"

This instructor responded to requests for answers with questions aimed at getting the students to discover the reasoning behind the problem for themselves. In doing this, the instructors perceived themselves as modeling for the students the type of questions that are asked in the problem solving process, as well as emphasizing that the students should ask these questions of themselves.

The instructors observed that many students were not used to this self-directed learning process; rather, these students were accustomed to learning mathematical knowledge directly from an instructor, without asking or even considering why the facts were true.

> People's beliefs about mathematics often come from what people in mathematics have told them. It was kind of like, "We know, and we will tell you and we don't question this really." If someone is an authority, then "Why?" is not necessarily a pertinent question. Like, if I just say, "This the way that it is." And I say it like that, and if you were my student, you probably don't say, "Why is that so?" In fact, it might not even occur to you that "Why?" is an appropriate question. Suppose now that I am the instructor and that I go far the other way and I don't commit at all. Now, although I make it clear that they are not done, if they are not done, then "Why?" comes not from me, but from the logical consistency of the mathematics itself, and through them.

This instructor stressed that in order for students to be independent problem solvers, they must learn to ask themselves the kinds of questions that reveal the reasoning behind the problem. By "stepping back" and refusing to provide the problem-solving methods and reasoning for the students, the instructors felt that this created a situation which compelled the students to "step forward."

<u>a. Strategies in giving help</u>

Our observations of the course and interviews with instructors revealed that not all instructors shifted responsibility to the students in the same way or to the same degree. This was indicated by the amount of assistance the instructor provided to students in the problem solving process. In this section, we will discuss the types and amount of help given to students as they worked on problems.

Giving background information

Some instructors gave assistance to alleviate some of the students' frustrations with adapting to a non-lecture format. The following instructor's comments illustrate his/her realization that many students were frustrated because they perceived themselves as lacking the tools to solve the problems.

> Well, the main other frustration is the feeling that has surfaced several times -- they don't have the tools. They want me to have presented the tools for them and have given them a little bit of a base. There was some feeling at first that I wasn't doing any teaching, you know?

Many instructors reacted to student frustrations by giving students the background information, in the form of definitions and examples, to solve the problems.

> R: Well, I only sort of gradually - just when I felt there was extreme frustration, I would say, "Okay, let's come together and let's just talk a little bit about what these ideas mean first, and then we'll tackle the problems."...I attempted to see if one couldn't do half a class of introduction or discussion. Let's suppose you're going to spend the week on a topic, so let's take half of one class, do that, and get them started on the problems the other half of the class. The next class do completely in groups.
>
> I: Oh, so what you're saying is that the first class, or half of that first class, you would lecture?
>
> R: Yeah, I would present the basic definitions of what we're talking about. I would pretty much say what's in the notes plus give a few more examples of things and let them ask some questions at that point, make sure they understand the definition. So that's really all I mean.

Some instructors also provided some guidance to the students in the form of giving definitions and the objective of the problem.

> And my own feeling is that the definitions and that kind of thing, one should give them. And, explain what it is that you are looking for. I tried to do it very carefully in a way that

> doesn't tell them how to go after it. But it does help them to understand, or formulate in their mind what it is, what objective they are trying to get to.

The instructor continued by commenting that the students should be exposed to as much of the problem solving process as possible, but should be guided, if necessary, in the direction of the solution.

> I find myself fitting a little more into, what shall I say, letting them do it themselves as much as possible, but giving them, maybe even little subconscious signals, as to the right direction, partly with the idea that I'm not sure that a long time spent in confusion is really contributing. That if they're heading in strictly the wrong direction, it seems to me the effort is really wasted and they might better spend the time discovering something else.

By providing a framework for solving the problems, examples and definitions, these instructors prepared the students to work productively toward the answer and also experience the problem solving process. As noted in the discussion on instructor goals (Tab 7, Section A.4), these instructors felt that it was important to channel the student learning process in order to ensure that they were exposed to a body of mathematical concepts.

Giving little or no help

In contrast, some instructors gave little or no explanation of the terms and concepts used in Math 130, and explained that giving help in the form of objectives, examples or even definitions contradicted their theory on how students learned mathematics. For these instructors, realizing the objective, finding the definition, and creating examples are fundamental aspects of the learning process. Learning mathematics involves not only formulating the answers, but the struggle that accompanies problem solving.

> I wanted them to wander around, because I want them to know that they can get from here to here on their own. If you just drag them up the straight and narrow, then when they get up there, they'll say, "Well, I got up here and I could never do that." But they can do this, they can wander around and get there.
>
> * * * * * * * *
>
> And I want them to realize that they always have to interpret a problem, come to an agreement on it. That is what the rules are, and if they are ambiguous, so be it. What you do in real life with an ambiguous problem is either, if somebody is asking you for a particular reason and the person is there on hand, you grab them and say, "Hey, look, this is an ambiguous problem. Which do you want me to do, this or that?" If the person isn't on hand, you do something somewhat general. You say, "Well, this is an ambiguous problem, but assuming this interpretation we can do this, and if not, there really isn't a solution, so we are going to now assume this." And I want them to take responsibility for a mature approach to, sort of possessing the problem first.

b. Viewing the responsibility for the answer: closure

The issue of providing closure, or an answer and method to solve problems, arose when the students did not arrive at a solution in their groups or in the class discussion. The instructors' view of providing closure was directly related to their conceptualization of the goals of the course. Some instructors considered closure compatible with the goals of the course and therefore did provide it. In fact, some instructors felt it was necessary to provide the answer in order for the students to learn mathematical facts and also to provide support. Others expressed that closure would prevent students from achieving the goals of the course and therefore did not provide the answer.

Providing closure

Some instructors explained that it was necessary to provide closure in order for the students to eventually see and understand the solution and as a measure of support. One instructor explained that s/he perceived the students as needing closure.

> Because, I think, maybe mistakenly, they are so much more unsure of themselves basically, that they need to feel like, "You're getting somewhere. You are learning something. You're getting better." They seem to want that anyway. I don't know if they need it, but it's really hard to resist. They're much more emphatic and persistent about asking you that!

Another instructor provided closure by putting a correct answer up on the board and challenging the students to figure out why it is true.

> I think it works very effectively, to get that answer up on the board. Get the right answer up on the board, and challenge them, on the question of, "How are we going figure out what is actually the right answer? Is there some way to do it?" And eventually, and I think it's fine, if the end of the period comes and you leave them hanging, and they don't know what the answer is. But I think the next day, when you come to it, it should eventually be resolved. If you can possibly avoid it, don't resolve it by fiat, but try to get it out of them, perhaps with a few hints, if they just don't seem to be getting it.

In addition to the students acquiring a correct solution, the instructor felt that this method helped to facilitate the students' shift into becoming independent problem solvers by creating a situation where the students also had to figure out the reasoning behind the solution as a class. For this instructor, this was advantageous in two ways: the students see the solution and also are involved in the problem solving process as a class.

No closure

Some instructors considered giving closure incompatible with their perception of the main goal of the course: getting students to be independent mathematical thinkers. These instructors explained that in order for students to be motivated and to learn, they should be responsible for

finding the solution.

> If they know that I am going to tell them at the end what the answer is, "Well, we'll just wait, and at the end of the class [the instructor] will say, 'Oh, this is what you are supposed to get out of the problem, here is a little summary, here is why it all works, and on to the next problem.'" I think that will also interfere with them taking ownership of them. Working hard to find out why....So that means that they really get interested; if you work hard on a problem from scratch, you become very interested, you become trained to look for that and you can't help but ask.

For this instructor, transferring the responsibility for getting the answer to the students is a fundamental aspect of the larger whole of the learning process. Students' motivation for working on the material is attenuated if they know they will get the answer from the instructor. According to these instructors, for students to be fully engaged in the problem solving and learning process, they must be left alone to discover the solutions to the problems.

D. Transferring the Philosophy of Math 130

In this section, we will discuss the instructor's views as to whether the underlying philosophy and methods of Math 130 are currently being transferred to new instructors, and how such a system of beliefs could be transferred. In addition, because the philosophy and methods underlying Math 130 differ from traditional lecture courses, we will also discuss the types of experiences that instructors new to the Math 130 method faced.

1. The need for a formalized way of transferring the philosophy of Math 130

Several instructors were concerned that the philosophy of Math 130 was not being communicated to new instructors. One instructor commented that the "how," or implementation issues of Math 130, was transferred rather than the "why," or the philosophy behind the course.

> When we talk to people about what we do, we talk to people more about how we do it, and not why we do it. And I'm wondering if the why is missing. That's the thing I worry about. Is the "why" missing? Will that have an effect? I can see that [responding to student questions about and frustrations with this new format] is one place where I see it might have an effect, and it worries me.

This instructor continued by identifying the link between the philosophy underlying Math 130 and its structure.

> Out of a philosophy you could create a "how." A "how" is available, there's reading to be done on different things. And I think the course could be done in different ways, in different structures, and it would be a great course. So, that's not the most important thing, how big the groups are, or whether you work first and then have a discussion about it, or whether you have them work on problems at home on their own. But I do think that what's important is that

> they are actually doing math itself. And not hearing it from you and then just applying it to different problems. But, how that's accomplished, it's not important.

Thus, for this instructor the implementation issues need to be grounded in the underlying philosophy of Math 130. The philosophy is not inherent to the structure and therefore if discussions about the course only center on the "how," then the philosophy may be disregarded, or even lost.

Some instructors considered the knowledge of, and belief in, the philosophy an essential part of convincing students that the method used in Math 130 is a better way to learn. According to one instructor, knowledge of the philosophy would enable instructors to respond to student queries about the reasons behind this new teaching and learning format.

> One of the things I'm kind of worried about, when students had concerns, I had to wait to answer them based on my philosophical beliefs on how people learn, and how the method, if you want to call it that, how it works. So I had a lot of beliefs about how students learn, etc. that I can use to talk to students, and I worry about people who come in to do it who don't have that. How they will adjust to such things. I think it would be even harder, I mean, how do you answer a question like, "Well, why are we doing this? And how can you expect me to know this if you haven't taught this to me?" And, "Aren't we taking forever to do this problem?" And whatever they see, or whatever they feel, they say things, not directly, but they make you believe they are thinking those things. If you don't have beliefs about why you're doing what you do, if you're just doing it because, "Okay, I'll try it. I know the other course wasn't working so well, so I'll do this." then how do you direct that, I'm curious to know.

For this instructor, properly answering these types of questions is critical in providing support and orienting the students to this new teaching and learning environment.

In addition to not being able to fully answer students' questions, some instructors indicated that without some knowledge of the philosophy, new instructors might not facilitate the students transition into learning the material in this new format. In the following interview excerpt, an instructor discussed this concern.

> Well, the problem you see -- If somebody doesn't go at it with that philosophy, it could easily happen that the it would get to the point where the teachers would find it easier to just say, "Here is how to do this type of a problem" and then, "Now sit down and do today's problem." Instead of not giving them any idea and then letting them figure it out.

An instructor who is unaware of the philosophy, or is ambivalent about it, might find that this method is too difficult to enact and return to the instructor-dependent format, in which the instructor provides the students with the method of solving the problem.

> What I'm afraid of is sliding back to a situation where you get faculty going in there who don't have this point of view and aren't willing to take the time to understand the course and who are simply going to do it their old way.

Thus, for some instructors, transferring the beliefs of Math 130 is crucial for it to continue in its present form. Moreover, these instructors felt that without any formalized way of reviewing and discussing the philosophy, new instructors may not have a context to understand the underlying reasons for the format of Math 130.

2. Methods of transferring the philosophy of Math 130

In this section we will present some instructors' views on how the philosophy of the course should be transferred. One instructor observed that a guide should be written that would discuss this aspect of Math 130 as well as provide more detailed information on implementation issues such as comments on those things the instructors found successful and unsuccessful about certain problems.

> And so, I think some kind of an effort if one is going to keep it to be this kind of a course some sort of an effort needs to be made, and that's why I thought maybe some kind of a teacher's guide. Now it's a lot effort to put something like that together, but maybe if those of us who taught different things, at least put in our comments where we thought they ought to be put in.

Another instructor commented that any teaching guide must be accompanied by other strategies for informing instructors about the methods of the course.

> There is a bit of a continuity problem. Namely, some instructors [in a similar course] are thrown in there with a booklet, but the booklet doesn't guarantee that you read it. And even if you intend to follow the materials, not having live role models makes it much harder to tell what is meant. So, we're attempting to provide serious continuity here, by having the people who are already slated, or likely to teach this, in the future, now attending these meetings and visiting the classes. And furthermore, when new instructors teach the course, we plan to sit in on their classes. So we get two-way feedback. Which, every time I've seen a program in which instructors get this kind of feedback, it has just always worked fantastically well. People get good at teaching in a certain way much better when they are watching model teachers and being watched themselves.

Many instructors were in agreement that instructors new to Math 130 would benefit from seeing someone model this new method and from having the opportunity to exchange ideas

and concerns about their courses. In fact, an instructor who had not taught the course before commented that such a dialogue between other instructors would have been helpful.

R: The lecturers in 130 will probably get together, and that will be helpful.

I: Why would that be helpful?

R: To see what problems they're having and what satisfactions too, and also to sort of compare how much of explanation is any one of us doing or how little or something like that.

I: So that you'd see the differences and similarities?

R: Um-hum, what you're facing.

I: How you're teaching also, then.

R: Right. I think maybe what would be helpful is at the end of the course, or on a continuing basis, if we made it clear to ourselves what the goal is at the end that we want them to be able to do.

I: Okay, what the goal is in regard to what?

R: Well, do we want them to understand these mathematical concepts plus reduce their fear of problem solving, and if so, what are the mathematical concepts that we want? Or is it mainly to reduce their fear of problem solving and exposure to the variety of math topics that there are? If there isn't agreement, I don't know if that's terrible necessarily.

This instructor observed that a continuous dialogue between the instructors might prepare them for unexpected issues that might arise, keep them focused on the goals of the class when dealing with the students adaptation to Math 130, and also provide support for the instructors themselves in their own transition into this new teaching paradigm.

3. Instructor experiences in the new teaching and learning environment

Many instructors lacked experience in teaching in the Math 130 format and were unsure what to expect. In this section we will discuss the experiences of the instructors teaching Math 130 for the first time.

The transition for instructors

In addition to facilitating the students' transition into the new non-lecture learning environment, many new instructors also described their own experiences of transition into a

new style of teaching. One instructor described this transition metaphorically as driving a car with different controls.

> Well, we all build up our reflexes and ways of doing things, and you have to change a lot of those. To me it's a little bit like suddenly having to drive a vehicle which has quite different kind of controls, and it's a different vehicle. There's a very definite learning process going on, and you have to build up other kinds of reflexes.

This instructor highlights the significantly different experience of teaching within this new method and the learning process that must accompany it. In the traditional classroom format, the instructor is viewed as having the role of dispensing knowledge, often times in the form of providing methods of solving problems and giving solutions. In Math 130, the instructors observed that they needed to alter their "reflexes" from giving the answer to guiding the students in the process of finding the solution. Many instructors recounted their own difficulty in making this transition to a new teaching style. One instructor described this transition as scary and risky.

> R: ... I experienced it as scary at first.
>
> I: Why scary?
>
> R: I experienced it like "I'm good at the traditional method. I can do it, but can I do this?" So it was almost like personal, like you don't want to fail at something. And now, it was personally scary, it was like a risk.
>
> I: ...Just because it was new, or because it was new in a particular way.
>
> R: It was riskier in a way that you don't get to plan out how it is going to go. You can get real good at giving a lecture and knowing what examples to show and what the high points are and what the two questions are that students are going to ask and everything will go pretty much the way that you planned. You can make it go that way when you are in control of the lecture. You can just ignore this, or say that we can talk about that tomorrow. It is plenty easy to make it go exactly how you want it to go. Anything can happen [in this new teaching and learning environment].

Another instructor commented that the transition into a non-lecturing teaching style was difficult for him/her because s/he enjoyed lecturing and also felt that one role as a teacher was to organize and present material to the students. This instructor described a "tension" between his/her enjoyment of a lecture-based teaching method and using a student-centered inquiry-based learning method.

> R: I miss lecturing, though, because I like trying to gather a topic together in a way I think is learnable, if that's the word, and I like to be able to present it in a way I think will be interesting. So that part I miss, so that's hard to - you never know whether my response to try

to introduce some of the basic ideas will satisfy my need or theirs! (laughs) Because that's part of the pleasure of teaching to me, is to take what I know and try to organize it in a way that it will be appealing to everybody else. On the other hand, I like seeing their discoveries, their feeling of power, and that's really fun, too. But then you have to stay out of it if they're going to get that.

I: Right. That's interesting.

R: So it's a tension. It's a tension. I don't know how much is just tradition, the fact that we've always lectured so that's what we should do. So that's the main thing that I'm missing, so neither of those is a very good reason to do it. (laughs) Or whether there isn't some good reason to do it because I'm the one that knows the material. Shouldn't I have some obligation to - isn't that what teaching is, partly at least, to be able to pass it on to somebody in a way that is exactly to them, at least partly?...One of your roles as teacher is - I mean, you're a teacher, and the other person's a student because that way you know something they don't know, and you're trying to pass that on. Organizing the knowledge so that it's accessible to the student is part of your job. But I don't know.

Reacting to students' level of mathematical background

Some instructors who taught Math 130 for the first time were surprised at the students' low level of mathematical background.

It was amazing to me how hard the students found the basic problems. That just surprised me. I didn't expect that. Even though they said they hadn't had set theory, maybe I didn't believe them. I thought, they must have had elementary set theory, that's impossible! So I was unprepared for that.

One instructor commented that knowing and perhaps expecting that the students entering Math 130 had very little background might have prepared him/her for students' reactions to the new method.

R: I wasn't prepared for their relatively low level of knowledge, I guess. Which isn't to say they're not bright enough, well, the set theory is a good example. I could think of others. I was surprised that they didn't have the basic concepts.

I: How has that affected, then, your method of - your role as an instructor in this class? If you weren't prepared for that, then how has that affected what you've done?

R: Well, we spent longer on topics than I originally intended. I've done more explanation in class than I would have thought.

I: Do you link the two of those, then - that you've explained more than you originally thought you would because of that fact, that they were less prepared?

R: Oh, yeah, sure. And because they seemed so upset by the fact that they were less prepared. Now maybe if I had talked to [another instructor] and [s/he] had said, "Oh yeah, a lot of them don't know beans about set theory, but don't worry about it, they'll be fine, just let them struggle," and maybe if I had watched them struggle, maybe then. And maybe by the end of this term I'll see that a lot of it was just sort of panic on their part, and they actually could do better than they thought. But a lot of them did have trouble with the set theory part.

E. Curriculum Issues

In this section we will discuss how some instructors viewed the curriculum and its effect on facilitating problem-solving skills and student learning of mathematical facts. Although the course was structured into sections of learning, the depth of the topics covered was flexible. As one instructor noted,

> One of the philosophies behind the course is that the curriculum, which is a nominal list of topics that they are going to cover, is flexible.

This flexibility allowed for the instructors to respond to students' needs quickly and effectively. The following instructor commented that s/he could discern better what the students knew in this new teaching and learning format, and thus could utilize the flexibility in the curriculum to deal with problems with student understanding.

> In the course of the conversation, there is stuff that in a normal setting you wouldn't see if you are lecturing. Like, "Oh, God, Amy doesn't know how to add fractions." It comes up, and you see it, but you wouldn't see it in general during a lecture. You might see it on a test and say, "I can't believe that these students are so bad." But not really get it in time to do something about it. You can make a decision then, "Well, should we stop and perhaps talk about this? Or is this too off topic?" and make up a sheet later that will address this issue and add that into the content. And I don't have any qualms about doing that, saying, "We need to talk about this. Forget what we're doing tomorrow, now we are doing this."

However, one instructor commented that although it was important for the curriculum to be flexible, it must also focus on learning specific mathematical facts.

> I do feel that there should be a body of knowledge on the whole that you're transmitting. It shouldn't just be a bunch of techniques actually. I don't think it should be a class just in problem solving itself or, well, if it is, then that's one thing, but if it's a class in mathematics itself, then there needs to be a certain amount - and there is some knowledge there, but it's so many different things.

This instructor also expressed concern that the curriculum was too fragmented; students were not learning material deeply and therefore were not feeling a sense of mastery of the material.

> But the reason [teaching Math 120 by the old method] wasn't very satisfying is that -- and this may still be a problem even with the current approach [the Math 130 method] -- is that because

> it's a number of different topics. I'm not sure that these students ever feel a real sense of mastery of anything in the way that calculus students can see themselves progress very nicely.

If the curriculum built on itself, the students could, in addition to developing their problem solving skills, feel a sense of mastery of the material.

> I'm speaking partly because -- that is, the disappointment, if it is that, is that I can see a lot of them could master some things pretty well, and I think that's a great feeling of success when you do that, and it builds confidence.

In summary, many instructors explained that the flexibility of the curriculum allowed them to react swiftly to the students' mathematical weaknesses. However, a few instructors felt that the curriculum, while flexible, was only superficially covering areas and students were not feeling a sense of mastery of the material. To correct this, a few instructors expressed that the curriculum should be modified so that it remains flexible yet builds on itself.

Appendix A:
Interview Protocols, Fall 1994

QUESTIONS FOR THE FIRST INTERVIEW FOR MATH 130 STUDENTS

1. What year are you in school?

2. When will you start student teaching?

3. What grade do you eventually hope to teach?
   [probe about why this is their preference]

4. Looking back, what do you think were the most important factors in your decision to become a teacher?

5. Was there anyone in particular that influenced you in your decision to become a teacher?

6. What subjects are you excited about teaching? What subjects are you concerned about having to teach?

7. Can you describe your math experience in middle school and high school? [probe for their "relationship" to math and what factors contributed to it]

8. What are you hoping to get out of this course?
   [Follow up questions: How do you feel about taking this course? Have you heard anything about this course?]

9. What are your impressions of the course so far?
   [probe for whether they like the course, don't like it, etc.]

10. What do you feel about working in groups in this class?
    [probe about their previous experience working in groups]

SECOND INTERVIEW QUESTIONS FOR CURRENT 130 STUDENTS
(edited 11/16/94)

1. Is the way that your are learning the material in this class different from other math classes?

2. What is the role of the instructor in all this?

3. Suppose that you're given a problem that you don't immediately know how to solve. What do you do?

4. What do you think of the group work?

5. How do you feel if you're working in your group, and you just can't solve the problem?

6. How do you usually go about solving a problem, from when you first see it to when the class moves on to another problem?
   [Find out what kinds of questions, if any, are still left open when the class moves on.]

7. What do you think of the write ups (individual and group)?

8. What do you think of the other writing assignments?
   [Get some examples.]

9. What is the atmosphere of the class? Do certain people talk a lot more than others? Do you ever feel uncomfortable in the class, or wish that it was different?

10. How do you feel about not having a text?

11. What have you thought of the exams?

12. What do you think of the grading?

13. Have your feelings about the course changed during this semester?

14. [If not answered elsewhere]
    What is the most useful part of the course?
    What is the least useful part of the course?

15. When you were in high school, did you feel that you understood math?

16. Is your understanding of math different now?
    [If so, probe as to why - subsequent courses, Math 130, etc. If not, probe as to why the math they've had since then hasn't helped.]

17. Have your feelings about math changed this semester?

18. Has this course had any impact on the way that you plan to teach?

19. How would you handle it if you were teaching in an elementary school and a student solved a problem, but not in the way that you were looking for?

20. Why do you think that this course is required?

21. When you think about the course a year from now, what will stand out?

QUESTIONS FOR MATH 120/130 ALUMNI WHO ARE STUDENT TEACHING
(edited 10/21/94)

BACKGROUND

1. What year are you in school?

2. When will you be finished with school?

3. What grades will you be certified to teach? Will you be certified for any additional teaching (i.e. special ed)?

4. What grade do you eventually hope to teach?
[Probe about why this is their preference.]

5. Do you want to teach any special courses (i.e. special ed, specific subjects in middle school, etc.)?

6. Looking back, what do you think were the most important factors in your decision to become a teacher? Was there anyone in particular that influenced you in your decision to become a teacher?

7. Most of the people in elementary education are women. How do you feel about that? Have you noticed any differences in how you teach or in how students, parents, or administrators relate to you because you are a woman/man? [Probe this - why do they think this is? Does it affect the teaching dynamics at all?]

MATH BACKGROUND

8. How good are you at math today?

9. Can you describe your math experience in elementary, middle and high school? [Probe for their "relationship" to math and what factors contributed to it] [Follow up questions: Did you do well? Did you enjoy math?]

10. When you were in high school, did you feel that you understood math?

11. Is your understanding of math different now?
[If so, probe as to why - subsequent courses, Math 120, teaching math. If not, probe as to why the math they've had or taught since then hasn't helped.]

12. Do you think that some mathematical training is needed for undergraduates in general?

13. Do you think that some mathematical training is needed for people in your certification program? Describe what you would like the education program to provide.

STUDENT TEACHING

14. What grade are you student teaching?

15. How do you feel about student teaching?

16. What things have helped you the most in preparing to become a teacher?

17. What subjects do you like teaching? What subjects are you concerned about teaching?

18. When do you teach math?
[Do they teach math at a set time, or incorporate it into other areas?]

19. Describe a typical math lesson.
[Do they teach math in groups, using worksheets, discussion, computers, games, etc.? How do they teach mathematics?]

20. How many days are like that? [Do they do something different once a week? Does the pattern change often?]

21. Why did you decide to teach math that way?
[Classroom teacher, previous coursework, previous experience]

22. How do you handle it if students solve a problem, but not in the way that you were looking for?

23. What are you trying to teach the students when you teach mathematics?

MATH 120

24. What year were you in school when you took Math 120?

25. How was the course taught?
[Probe this question. Get many details.]

26. How did you feel about taking the course before you started?

27. How did you feel about the course when you had finished it?

28. Did your feelings about math or your own ability at math change during the time that you took the course?

29. Were there any parts of the course that you found useful?

30. Were there any parts of the course that were not useful?

31. What was your favorite part [area, material] of Math 120?
Have you taught that to your students at all?

32. What was your least favorite part of Math 120?
Have you taught that to your students at all?

33. Do you use any of the information that you learned in the course when you teach math? [Content or method]

34. Has this course influenced your teaching at all?

QUESTIONS FOR PEOPLE WHO HAVE HAD MATH 120/130 WHO ARE NOT TEACHING
(edited 10/21/94)

BACKGROUND

1. What year are you in school?

2. When will you be finished with school?

3. What grades will you be certified to teach? Will you be certified for any additional teaching (i.e. special ed)?

4. What grade do you eventually hope to teach?
[Probe about why this is their preference.]

5. Do you want to teach any special courses (i.e. special ed, specific subjects in middle school, etc.)?

6. Looking back, what do you think were the most important factors in your decision to become a teacher? Was there anyone in particular that influenced you in your decision to become a teacher?

7. Most of the people in elementary education are women. How do you feel about that?
[Probe this - why do they think this is? Does it affect the classroom dynamics at all?]

MATH BACKGROUND

8. How good are you at math today?

9. Can you describe your math experience in elementary, middle and high school? [Probe for their "relationship" to math and what factors contributed to it] [Follow up questions: Did you do well? Did you enjoy math?]

10. When you were in high school, did you feel that you understood math?

11. Is your understanding of math different now?
[If so, probe as to why - subsequent courses, Math 120, teaching math. If not, probe as to why the math they've had or taught since then hasn't helped.]

12. Do you think that some mathematical training is needed for undergraduates in general?

13. Do you think that some mathematical training is needed for people in your certification program? Describe what you would like the education program to provide.

STUDENT TEACHING

14. When will you be student teaching?

15. How do you feel about your student teaching? [Are they nervous about it, looking forward to it?]

16. What things have helped you the most in preparing to become a teacher?

17. What subjects are you excited about teaching? What subjects are you concerned about teaching?

18. How do you plan on teaching math?

19. Why did you decide to teach math that way?
[Classroom teacher, previous coursework, previous experience]

20. How would you handle it if students solved a problem, but not in the way that you were looking for?

21. What kinds of things will you try to teach the students when you teach mathematics?

MATH 120

22. What year were you in school when you took Math 120?

23. How was the course taught?
[Probe this question. Get many details - was there a text?]

24. How did you feel about taking the course before you started?

25. How did you feel about the course when you had finished it?

26. Did your feelings about math or your own ability at math change during the time that you took the course?

27. Were there any parts of the course that you found useful?

28. Were there any parts of the course that were not useful?

29. Will you use any of the information that you learned in the course when you teach math?
[Content or method]

30. Has this course influenced your teaching at all?

Appendix B:
Interview Protocols, Spring 1995

THE FIRST INTERVIEW FOR MATH 130 STUDENTS
(revised 1/25/95)

1. What year are you in school?

2. Have you been admitted into the school of education?

3. When will you start student teaching?

4. What grade do you eventually hope to teach?
   [probe about why this is their preference]

5. Looking back, what do you think were the most important factors in your decision to become a teacher? Was there anyone in particular that influenced you in your decision to become a teacher?

6. What subjects are you excited about teaching? What subjects are you concerned about having to teach?

7. Can you describe your math experience in school?
   [probe for their "relationship" to math and what factors contributed to it.]

8. What are you hoping to get out of this course?
   [Follow up questions: How do you feel about taking this course? Have you heard anything about this course?]

9. Why do you think that this course is required?

10. What are your impressions of the course so far?
    [probe for whether they like the course, don't like it, etc.]

11. What do you feel about working in groups in this class?
    [probe about their previous experience working in groups]

12. Suppose you are given a problem you don't immediately know how to solve. What do you do?

13. How good would you say that you are in math? [probe]

14. How do you feel about not having a text?

SECOND INTERVIEW QUESTIONS FOR CURRENT 130 STUDENTS
(edited 4/21/95)

1. Is the way that you are learning the material in this class different from other math classes?

2. What is the most useful part of the course?
   What is the least useful part of the course?

3. What do you think of the group work?

4. When you are in your group, how do you usually go about solving a problem, from when you first see it to when the class moves on to another problem? [Find out what kinds of questions, if any, are still left open when class moves on.]

5. Has there been any time that you have not been able to solve a problem? How do you feel about that? [How is it handled?]

6. What is the role of the instructor in this course?

7. Describe the atmosphere of the class? [Probe: Do certain people talk a lot more than others?]

8. How comfortable do you feel asking questions in your group? in the class discussion? going to the board?

9. What do you think of the grading?

10. What have you thought of the exams?

11. What do you think of the write-ups (individual and group)?

12. What do you think of the other writing assignments? [get some examples.]

13. Have your feelings about the course changed during this semester?

14. When you were in high school, did you feel that you understood math?

15. Is your understanding of math different now? [If so, probe as to why - subsequent courses, Math 130 etc. If not, probe as to why the math they've had since then hasn't helped.]

16. Have your feelings about math changed this semester?

17. Has this course had any impact on the way you plan to teach?

18. Why do you think that this course is required?

MATH 130 INSTRUCTOR INTERVIEW
REVISED 3/8/95

CONTEXT:

1. History of teaching here at Madison.

2. Why did you become interested in teaching Math 130?
[How did you get involved?]

3. How would you describe your approach to teaching Math 130?

4. Is this course different from the way that you have approached other courses you have taught?

4a. If yes, how do you experience the difference? What has the transition to this method been like (for you and for the students)? [Can you give a specific example?]

4b. If yes, is there anything that has been, or would be, helpful for you in making this transition?

5. What are the skills that you want students taking Math 130 to acquire? What are your teaching goals?

6. Do you have any students who have concerns about the course? If so, how would you address their concerns? [Do students ever question you about why you are having them learn the material this way (ie why you won't just tell them how to do the problems)? How do you respond to them?]

7. What is the role of the instructor in this course?

8. How do you respond to students' requests for the "correct" way to do a problem (and the "right" answer)?

9. How do you form the student work groups in your class and how often do you switch the groups?

10. Can you describe what you would consider a good class day for Math 130? A bad class?

11. Are the Math 130 students being taught in a manner in which you hope they will teach their students?

12. How do you assess the student learning in the course?

13. Do you meet with other faculty teaching this course and have you observed each other's classes?

14. Will your experience in this course influence your teaching in other courses?

Appendix C:
Sample Classroom Observations

Section 1 Observations

12:07 I arrive two minutes late. The students are in rows facing the front. The instructor begins the class by forming new groups (up until now the students had stayed in the groups that they originally formed). S/he uses part of a deck of cards. Each person draws a card, and the four people who get the same number form a group. Two people are missing, so the instructor assigns them to two smaller groups.

Two of the new groups are in almost exactly the same composition as they had been. Three women who were in a group together [and, as the instructor tells me later, the most upset about changing groups] draw the same numbers, and they are obviously happy about that. They tell their previous fourth member (now in a different group) that it is too bad that she isn't in their group again.

There is one person in the class who, like me, seems to be observing the course.

All of the groups are talking.

12:15 The students get a "number" worksheet with four problems on it:

1. Every number is one less than some number.

2. There is a number which is not equal to the square of any number.

3. There is a number which is equal to the product of any two numbers.

4. There is a number such that for any number there is a third number you can subtract from the second number to get the first number.

At the bottom of the worksheet it states that for this assignment "number" means natural number 0, 1, 2, . . .

The students are to discuss which of the four statements are true, false, or ambiguous for the natural numbers.

People discuss the worksheet. One group of four women discusses the meaning of "any number" and "a number" in the worksheet.

12:18 The group that was so excited to remain together get a little off task and talk about one of the women's calculator. She says that she loves it for physics.

12:20 Everyone gets into a large group.

The instructor takes a poll about the whether the four statements are true, false, or ambiguous. S/he has people raise their hands and s/he records the results. The instructor tells them that afterwards they will talk about their conclusions, and if everyone agrees about a question s/he will argue the other side.

The results are:

	TRUE	FALSE	AMBIGUOUS
1.	many		few
2.	many	few	few
3.	many		one group, because they aren't sure of the question.
4.	some	some	most

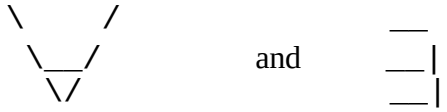
12:22 They argue it out for the first statement. People aren't sure if "some number" refers to a certain number or if it means "any number." The instructor asks if both interpretations are reasonable, and people argue over whether the ambiguous interpretation is OK. One woman says no, that is reading too much into the problem. Another says that it depends on how you read it. The instructor says that there is no right answer. It depends on English language usage. S/he tells them that's why they'll talk about math language.

12:27 They discuss the second statement. One person explains why they think that it is true. Another woman explains why she thinks it is false. A third woman says that she things the second woman misinterpreted the statement, and she explains it to the second woman.

12:30 They discuss the third statement. Most of the people think that it is true - essentially that when you multiply any two numbers you get a number. The instructor argues for the other side. S/he asks them, "What is that number? If there is a number that is equal to the product of any two numbers, I want to know what that number is." People seem a little confused, but then at least some of them seem to recognize the ambiguity.

They do not discuss the fourth statement. The instructor says that some of the statements on the sheet are ambiguous, and some are not.

12:35 They move on to another topic: the symbols



[in these notes, I will write these and \/ and =| respectively]

The instructor tells the class that you read the statement

\/x, x < 2x

as "For every x, x is less than 2x"

One woman asks what the difference is between the statements: "\/x, x < 2x" and "x < 2x"

The instructor explains that "x < 2x" may be true for some x, and false for others, while the statement "\/x, x < 2x" is either true or false.

The instructor also stresses that you need to specify the universe that you're working in (whether it's the natural numbers, the rational numbers, etc.)

The instructor describes the statement

\/x, \/y [x = 0 or x > y]

[S/he actually uses the logic symbol \/ for "or", since they have learned those symbols.]

A person asks for clarification - does s/he mean \/x AND \/y or \/x OR \/y. The instructor says, "Good question," and explains that it means \/x AND \/y.

The class discusses the truth of the statement. One woman gives the counterexample where x = 3, y = 4. The instructor mentions that a single counterexample is all that is necessary since the statement says that it should hold for every x and for every y.

12:43 S/he now moves on to explain =|, which means "there exists . . . such that" or "there exists . . . for which"

He gives the example

=| u, u = u^2 in N

which is read, "There exists a u such that u = u^2"

12:45 The instructor explains that the statement

\/x, =| y y > x

is different from the statement

=| y, \/x y > x

A woman asks if this is still in the natural numbers. The instructor responds, "Good question," and says that yes, it is.

S/he tells them to remember this for Monday.

12:46 The instructor tells the class, "Some people accuse me of lack of closure," and a woman says jokingly, "Us accuse you? Never!" S/he tells them that s/he has decided to spend the remaining time letting one woman give her answer to a problem they had worked on earlier. She explains her solution.

12:50 Another woman explains how she solved the problem, but she isn't sure if her answer is right [it's certainly a right idea, I notice.]

12:55 The class is over.

The instructor looks over my notes later in the day. S/he tells me then that the class asked a lot of good questions today. S/he says that the students are starting to be more analytic about their questions. They have also reached a point where when s/he tells them to spend a few minutes thinking about something before the next class, they do spend a few minutes thinking about it.

Section 2 Observations

1:15 The instructor writes on the board:

Please write the following information on a piece of paper and turn it in to help us gather statistics.

1. What degree program are you in?
What year in school are you?
When did you enter the degree program you are in?
[If you are not in a program yet, give expected date].

2. What math classes have you taken in college?

3. What math classes are you planning to take?

4. What math classes are you planning to take based on the specific program you are in and the date you entered the program?

1:20 The instructor tells people to copy down the information.

The chairs are haphazard, presumably left that way from [the previous Math 130 class] the hour before. The people [it turns out later] are sitting roughly in their small groups. The instructor walks around the front, and gives one woman in the back a sheet from last Wednesday.

1:24 The instructor stands at the front of the room and asks, "Have any groups got an answer to problems 3 and 4 from last time?" [The Locker Problem Sheet]. No one answers. S/he says they'll work a little more and then get into a big group and talk about the problems more generally.

The instructor collects papers from everyone (from the questions on the board), and the people form into groups: two women and a man; two women and a man; a woman and a man; three women; two women (later joined by a man); two women and a man. The two woman and man near me talk about the education program. One woman says that if she doesn't get in, she'll wait a year and reapply.

1:29 People talk in their groups. In the group near me, a man asks if there's a limit to the number of factors a number can have. One of the women says no, because you can go 2*2*2*2*2*2 etc.

1:35 A man walks in a joins the group of two women.

The instructor walks around and looks at the groups. A woman in the group in the far corner from me talks about how hard it is to study for exams.

1:40 The instructor says that they'll talk in a large group now. The man near me turns his chair from facing the back (where the rest of his group is) to facing the front. No one else moves. The chairs are still jumbled. The instructor stands at the front to talk.

The man in the group near me says that his group was wondering if, since there is no limit to the prime numbers and no limit to the positive integers, can you go as far as you want in terms of the number of factors a number can have. This is the same question that he asked of one of the women in his group, and she was convinced that the answer was no, and explained why. The small group didn't talk about it much after that. [Is he looking to the teacher for verification?] The instructor indicates yes.

The class looks at an example: 7^5 * 11^3. The instructor asks the class for factors. S/he gives "1" as an example. The class suggests: 11^3, 7^5, 7, 11, (pause) 7^2, 7^3, 7^4, (pause) 11^2, (pause) 7^5*11^3.

As they suggest these, the instructor writes them in the form of a table:

1	7	7^2	7^3	7^4	7^5
11					
11^2					
11^3					7^5*11^3

A woman says, "I see. You made a table." The class fills in the rest of the table.

The instructor asks, "What other factors do I have besides these?" No one answers. S/he asks, "If something divides this number, what kind of factors does it have?" There is no answer. S/he asks, "Is 2^3*3^5 a factor?" There is a pause, and then a man from the small group with just two people says no. The instructor asks, "What about 7^6*11^2?" There isn't an answer. S/he pursues this with "What can you multiply that by to get 7^5*11^3?" After another pause, s/he asks, "Is it possible or isn't it?" One woman starts to say, "You can't." and a man starts to answer.

The class agrees there are no more factors than the table. In response to how many factors there are, a woman says 4*6.

1:45 The instructor asks how many factors 13^5*17^3 has. A woman says 6*4 and explains that you go one more than the exponents. The instructor looks at her and asks about p^a*q^b. She says (a+1)*(b+1).

The instructor asks, "Does anyone disagree?" The class is quiet. The instructor asks, "Does anyone want this explained better?" and no one answers.

The instructor says to see if they understood, "Is there a number under 100 with exactly 10 factors?" A man says that his group found 80 = 2^4 and also 48 = 2^4*3

The instructor asks if there are any others. A woman says 2^4*7, but the instructor says that is over 100.

A woman suggests 2^4*2. People say that doesn't work, and the instructor says 2^4*2 is 64, which they know (from Wednesday) has 7 factors [in fact, 2^4*2 is 32, which has 6 factors, but no one points this out.]

A man (who'd spoken before) asks about an equation for p*q*r. The instructor looks at p^a*q^b*r^c. A man says he doesn't think you can do it, like for 30. Someone suggests (a+1)*(b+1)*(c+1). The instructor tries it for 30 = 2*3*5 and gets 2*2*2 = 8 factors. Then s/he lists the factors of 30 (the class names them): 1, 2, 3, 5, 6, 10, 15, 30. He says, "In fact, this is the right formula."

1:52 The class moves the Problem #4 on the Locker Sheet. The instructor asks a woman for her solution, and she says, "This is probably really wrong," and gives 964 = 72*12. The instructor factors it: 964 = 12*72 = 2^5*3^3. How many factors? 24

The woman next to me gives 960, with 26 factors. S/he asks how they got it, and she says the man did it. He explains that 60 had the most roots under 100, so they chose 960. The instructor has them write out the 9:60 = 2^6*3*5 (s/he has them give each step). A woman asks, "Twenty-eight factors, wouldn't it be?" She's right.

The instructor asks if anyone found any others, and after a short pause says, "It looks to me like this is as big as it gets." A woman asks how to get a lot of factors, and the instructor asks the class. Some people talk a bit among themselves, but the instructor talks to the class.

[Actually, 840 = 2^3*3*5*7 has 4*2*2*2 = 32 factors, so 960 really doesn't have the most. No one suggested any others, though.]

A woman asks if this is a write up and the instructor says yes, an individual write up.

2:00 The instructor passes out the worksheet "Life in Hell." One woman (far from me) leaves.

The group near me is confused. The woman asks how can you ever get out. They finally come up with starting at 1 and continuing 1, 2, . . . They are still confused, though, because the devil could have picked a number 90 digits long. The man agrees that there's no strategy, but why would they give this as homework if there were no strategy? They talk about it and decide to go with 1, 2, . . . and the woman says, "That's not a strategy." The man replies, "I know, but it's better than guessing randomly."

2:09 People start to leave.

After class, The instructor told me that the class was confused by factorization and s/he had led

them more than he wanted. I agreed that I'd been surprised that they paused on whether 2^3*3^5 was a factor of 7^5*11^3.

Section 3 Observations

2:25

On the blackboard, the instructor has written:

Folio pages due today.
Reflection on hard problem due Wed.
For *you*
--What do you see as benefits?
--Detriments?
--Will you assign them?

The students are seated in a semicircle with the instructor also seated at one end. There are eleven people: nine women and two men.

They are talking about the "apple" multiplication worksheet. People are not clear about what "closed" means, so they talk about it. The instructor talks about what "closed" doesn't mean and asks if people have other ideas.

As an example, the instructor makes up a set {1,2,3} with a "star" operation that is not closed:

_*|_1_2_3_
1 | 1 3 2
2 | 3 5 2
3 | 1 3 1

The students discuss what it is about this that makes it not closed. One woman suggests that it is because 1*3 and 3*1 are not the same. Another woman suggests that it's because 2*2 = 5 and 5 is not in the set {1,2,3}. Another woman suggests that it's because there are two 2s, three 1s and three 3s -- there aren't the same number of each.

The instructor says that these are all possibilities for why its not closed.

People continue to talk about it and the group agrees (as a whole, more or less) that 1*3 = 3*1 not being equal mean that the set is not commutative. The instructor agrees, and explains that it is the 5 that makes the set not closed under *. S/he asks someone to explain this in a different way, and one woman does.

2:34

The instructor asks for examples of sets and operations which are not closed. The class talks about this for a bit.

2:41
The class discusses associativity. They continue this discussion for a while. The instructor has people go up to the board. One woman gives an example (at the board) of the type a*b*a and starts to show how (a*b)*a = a*(b*a). She stops and changes to a different example of the type a*b*c explaining that her first example would automatically work because she knew that "apple" is commutative. I don't think that everyone heard or understood the explanation and the instructor didn't push that, but I was impressed. The woman's reasoning was entirely correct, and shows some real mathematical sophistication.

2:50

One man is confused about what (triangle "apple" circle) "apple" triangle means. Other people try to explain it.

The instructor asks if anyone can find an example in "apple" multiplication that is not associative. The students challenge him/her to find one, but s/he finally admits that s/he can't. They tell her (in good spirits) that she owes them a party.

2:55

The instructor asks if addition is associative. Is subtraction associative? After people talk about this a bit, s/he explains that they've given examples of associativity and this is <u>not</u> a proof that "apple" is associative. She asks how they could prove associativity. One woman says that if you showed *every* case this would be a proof. The instructor asks how many cases there would be, and the students say it must be something like 1 + 2 + 3 + 4. Someone else says it's probably 1*2*3*4. The instructor doesn't add anything, and they move on. The instructor says that if you know something more about the set, like if it's the set of real numbers, sometimes you can used that to show that it's associative. One woman says she doesn't really understand apple, so she can't prove it's associative.

3:00

The instructor has a man give an example of how subtraction is not associative:

(3 - 2) - 1 = 0 but 3 - (2 - 1) = 2

3:08

They talk about the identity. One woman give the example with subtraction that 9 - 0 = 9, but 0 - 9 = -9, so the identity doesn't work. People talk about this, and the instructor points out that while "apple" isn't too interesting, lots of people use addition, subtraction, multiplication, and division. Since they'll be teaching these concepts, they should have some understanding of them.

In talking about inverses, the instructor says that the "inverse is kind of personal." An inverse

goes with a particular element.

3:15

The students are still arguing over inverses. In apple, they wonder what the inverse of triangle is. [ed note -- this is the same thing another Math 130 class was wondering about]

The bell has run, but people are still arguing about the math. One woman asks if they can continue this on Monday, and the instructor says, "Yes."

Section 4 Observations

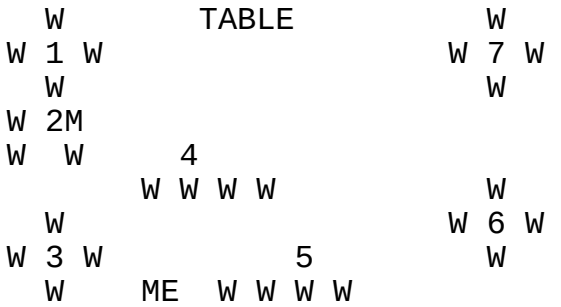
I walk in at 11:58. I overhear a male discussing when an assignment is due with two females. They tell him an assignment is due today and he says, "This class is stupid." One of the women responds, "That's because you didn't do it." He doesn't respond.

People appear to be sitting in groups. Most appear to be working on some problem or another, or discussing the problems, and/or socializing. Two students sitting in front of me discuss their homework grade and what the instructor had told one of them about it (previous to this class). The students to my left are discussing whether they had gotten into the school of ed.

The instructor comes in at 12:00 and erases the board. When finished with this, s/he hands out homework. When the bell rings, the instructor asks for the homework and walks around the room and picks them up from each person. The instructor tells the students to work again on the "condo" problem and students begins to discuss in their groups.

Observer's note: The instructor has not changed groups yet this semester. The students sit with their groups when they walk into the classroom. The group observation is discussed at the end of this paper.

The room is situated as follows:



The groups discuss this problem until 1:17, when the instructor asks the class if they have the answer. The class discussion seemed to follow a pattern. The instructor would ask a question, and would then call on someone in a group for the answer and their reasoning behind the answer. As this person explained, The instructor would write their explanation on the board. When they finished, he would ask the class if the answer and explanation were correct. Most of the time, students would raise their hands, and the instructor would call on them. The students would offer a different solution or input on the one just given. The instructor would again write this person's comments on the board.

Observer's note: The students were very active in the class discussion. For each problem, at least 3 of the 7 groups would give different solutions and explanations. At one point, all of the

groups came up with different solutions. Throughout this discussion, the number of people that appeared to be paying attention to the explanation of one of the students wavered between 1/2 and 1/4 of the class. The rest appeared to working/thinking to themselves or discussing with their group members.

After the instructor went through all possible solutions the students had come up with, s/he would then ask them if one was correct. At no time during this discussion did s/he say whether a solution was correct or incorrect. For example, after receiving 5 different solutions to a problem, a student asks if the one they just went over is correct. The instructor says, "Is it? People seem to think it is?" S/he smiles and then waits for a response. When no one talks, s/he says that they will then talk about the cyclist problem. A few students groan and seem a little frustrated that he did not say whether the solution was correct or incorrect. After the second time he does not give an explanation for a problem, one person in group three says, "This class is crazy!", jokingly.

At 1:36, the instructor tells the students to work in groups on the next problem. The students work in their groups until the end of the hour. When the bell rings, most people are packed up and ready to leave. At 1:56, all but 5 are gone. Three are waiting to talk to the instructor, and two from group 2 are discussing the problem.

Group Observations:
During the times when the students were working in groups, The instructor would walk around the groups. Except for a few cases, s/he would not initiate a discussion with the students, but would watch what they were doing as s/he stood close by. Students also did not ask the instructor many questions as he walked around.

Group 1:
This group appeared to mix working independently with discussions and explanations. Throughout the hour, there would be at least two students (not the same each time) discussing some problem and the others working independently or listening.

Group 2:
The male in this group appeared to direct his discussions only to one other female in the group. The female at the receiving end of this discussion did discuss a little with the other two, but not much. The other two students appeared to work independently throughout the hour.

Group 3:
At the beginning of the hour, the students discuss the subject differences between 131 and 130. This group appears to work together throughout the hour. Every time I observed them, at least one person was discussing a problem and the others were listening or working independently. At 1:15 the instructor walks by this group, and they tell him/her that they think they got the answer. S/he says, "okay." They laugh in response and say to each other, "S/he won't give us the answer!"

Group 4:

The setup of this group appears to dictate the nature of the discussion in this group:

W W W W

The two women on the left appeared to discuss the problems throughout the hour. The two women on the right appeared to work independently throughout the hour.

Group 5:
This group was set up like group 4. The students worked independently throughout the hour. There was almost no discussion between them.

Group 6:
This group appeared to work together similar to the way that Group 1 did.

Group 7:
This group appeared to work independently throughout the hour, with a few discussions between three of the group members.

Section 5 Observations

I walk in the room at 1:14. Five students are talking socially, doodling on their papers, or reading a book. The remaining 10 or so students appear to be working in groups in the workbook. The instructor begins handing out papers at 1:15. The class volume increases as students react and discuss their grades on the assignment. It appears that this is a group write-up. The group to my left received a 12 out of 12, and the group to my right earned an 7 out of 12. When the instructor hands the homework to this group, s/he tells them that they can redo the assignment if they like, because there were many mistakes.

Observer's note: I spoke with the instructor after class and s/he said that she had just switched groups two weeks ago which means that the students have worked in their new groups for 4 class periods. I should also note that I sat apart from the groups for this observation.

At 1:20, the bell rings. The instructor asks for their attention and the class quiets down. The instructor states that s/he received a couple of questions from students and would like to discuss them with the class. At first, about 1/4 of the students are looking up and appear to be listening. After about a minute, all but 4 appear to be listening. At one point, when the instructor asks a question to the class, about 10 - 12 people respond together with the answer.

From 1:24 to 1:30, the instructor discusses addition and subtraction in base 6. The instructor says that there is "no intuition about addition and subtraction in base 6." In other words, things aren't what they seem in base 6: 10 is 7 base 6, etc. The process that the instructor goes through in these explanations is to walk through a step, asking the entire class questions on the way. About 6 or so students generally respond by verbalizing the answer. At 1:25 a student raises her hand and The instructor calls on her; the student asks the instructor to do an example of division in base 6. The instructor asks the student to think of two numbers in base 6. The student does and the instructor puts in division form on the board:

$$3 \overline{)56}$$

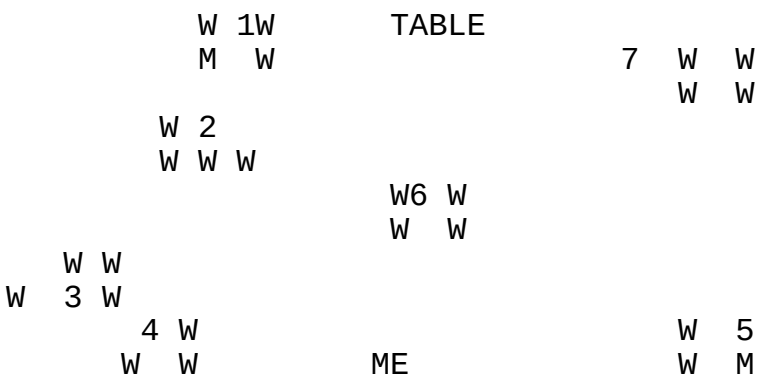
The instructor then walks through the division with the student, asking the student how she should do each step of the division. The student works with the instructor, responding for example, that the 3 goes into the 5 once, etc. The confusion seems to come with the remainder, and this is the only part where the instructor actually tells the student what is right or wrong; The instructor tells her that if a 2 is left over, to write R2 as you would in base 10 division.

At 1:30, The instructor has finished discussing base 6. The class starts to move into groups, although the instructor has not explicitly told them to. The instructor seems satisfied that this is what she wants.

Observer's note: I was confused at this point. The instructor did not say, "Move into groups."

I think the students took a clue from his/her behavior -- a halting of the group discussion, perhaps.

When the students have assembled into groups, the room looks like this:



Group Observations:

Group 1:
It appears that all of the students work independently; however, two of the students did discuss problems occasionally throughout the hour. The students that do talk are the male and a female. At one point, the aforementioned female made a statement. Two group members looked up, and the male commented something and pointed to some thing on her paper. The fourth student did not look up from her paper at all. At 2:05, one of the students pulls out a newspaper and reads it until the end of class. When the bell rings, this group has their coats on and bags ready and leave the classroom.

Group 2:
Observer's Note: The placement of the chairs in this group seemed to either indicate or reflect certain behavior:

W
W W W

The woman on the end appeared to work independently, but would talk with the student to her left. At one point, the third woman from the right listened in to the discussion, and then the fourth woman then listened. At one point, a woman in group 3 asked a woman in this group, "How are you doing?" The woman responded, "Good, but frustrated", and then looked on as the others worked. At this point, two of the students are working, while the other two look on. At 1:55, two students discuss how long it will take to do the problems. One says, "It will take forever to do." Another woman raises her hand and asks the instructor to come over. She shows the instructor the problem and asks her if it is okay. As she does this, the other three listen. When the instructor responds with a suggestion, all four listen. At 2:10, the group is ready to leave and does.

Group 3:
One student seems a little separated from the other three in the group, and this is reflected when they discuss problems:

W W
W W

Throughout the hour, this student seldom looked up when the other students discussed a problem; in addition, when the other three discussed something, none of them looked in this woman's direction, speaking only to one another. However, when the other students were socializing, this student did listen and made a few comments. For a couple minutes of the hour, this student sat in a relaxed pose with her arms folded, not working. When the bell rang, this group stopped working and left.

Group 4:
Two people in this group worked together throughout the hour. The third listened but does not say much. At 1:45, all three are working. The instructor walks over and looks at their work. S/he takes a pen and paper from one person and suggests that they try a simpler way and explains it. All three listen. One student explains to the instructor her interpretation of writing letters in base 6. The instructor says it is correct, and mentions that she should think of the numbers as place-keepers. The students ask him/her, "If we use base 10 in everyday life, why do we use base 12 for our clocks?" I didn't hear much of the instructor's response, other than our "hands and toes" affected how we add. At 1:55, the student who mainly listened to the other two, opens a book from another subject and begins writing in her notebook. The other two are still working on the problems. At 2:03, a student raises her hand and the instructor comes over. The instructor asks for a sheet of paper and suggests writing a simple table. Then the student writes down the table while the instructor looks on. The girl who opened the book still reads while the instructor is there. The instructor notices this and asks the woman who was doing the work to explain it to both students. The woman reading the book understands the hint, puts her book down, and listens to the explanation. At 2:10, this group packs up and leaves.

Group 5:
This is the group that the instructor said could redo their homework. Throughout the hour, this group would occasionally discuss how to redo this homework, and one person raised her hand twice to discuss the problems with the instructor. This group seems to work together throughout the hour. The male talked as he worked through the problems. Occasionally, another woman listened and discussed the problems with him. From 1:30 - 1:33, a woman talked with the instructor at the front of the classroom; it appears to be about the returned homework assignment. One woman explains to the other two how do work base 6. The male appears to understand, and helps to explain it to the last woman. The woman explaining it says, "It is hard to understand at first. You have to hunt and choose." The woman responds, "It kind of makes sense. I could never do it again." This woman seems to have trouble throughout the hour; later on, she asks the other two if she did problems 5 or 6 right. They look on and explain them to her. She responds, "I don't know if I can ever do it." The tone in he voice is not frustrated, it is stated as a matter of fact. At the end of the hour, this group makes no move to leave. They work until

2:13 and then leave.

Group 6:
This group appears to mix working independently, with discussions throughout the hour. One woman in the group raises her hand three times throughout the hour to receive the instructor's help. No other person in the group raises her hand. At the beginning of the hour, one of the members says, "It is better to work by yourself in this situation and then compare the answers." They then work independently. After about 10 minutes, they discuss the answers they have gotten. When the bell rings, this group remains working for a 2 minutes and then leaves.

Group 7:
At 1:30, the instructor is with this group after seeing someone raise her hand. The other three are not listening, and do their own work. One of the women discusses problems with the women to her left and right. These two women rarely discuss problems with one another. It seems that one will discuss with the central woman, and then the other will join in. The central woman and the woman on her left, seem to discuss throughout the hour. At times, the other woman on her right would join in the discussion. A few times, the two will ask this woman if they had done a problem correctly. The fourth woman worked independently throughout the hour. When the bell rings, this group works for another minute and then leaves.

I talked with the instructor after class and s/he told me that when s/he switched the groups, s/he put two people from the previous group together so that people would at least one person in their new group.

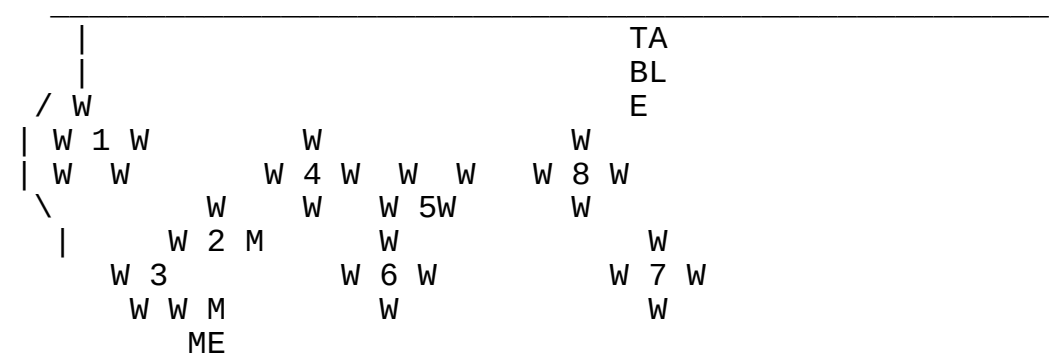
Section 6 Observations

I walk in at 12:56 and as I sit down, the instructor tells the students to get into their groups. A student responds by saying that they still have four minutes before the bell rings. The instructor then says, "Well, then you have four minutes to get together." The students start to get into their groups. This is the only time s/he tells them to get into groups. As students walk into the class, they sit down with their groups. In these four minutes, no one is looking at their workbook or anything that looks like a homework paper. Many students are talking to one another and two are reading the newspaper. The instructor hands out homework papers. The class volume is quite loud, with many people talking to one another.

At 1:00, the instructor raises his/her voice over the volume of the class and says that they will finish the fraction "business" today. The class quiets down in response. The instructor asks the class if they are prepared to talk about some problems from last time. A few say no, but most people do not respond. The instructor says they have 10 minutes to work on these problems.

Almost immediately, the volume increases as students get organized (open notebooks, workbooks to right page) and as they discuss the problems.

The room is set up like this:



Observer's note: During this class, I sat separately from the groups in order to get a larger picture of what goes on in the class. On this day, the class sat in their groups, but the focus of the class was mainly on a class discussion. The instructor would tell them to get discuss ideas in their groups for five minutes or so, and then would discuss them as a class. I am assuming that this day was unusual. I will discuss the group observation at the end of this paper.

At 1:10 the instructor begins to go over the problems with the class. S/he goes over each problem one by one, asking them what the answer is. Almost always, a student will verbally volunteer an answer without raising his/her hand. With that answer, the instructor will ask the class if it is true, or false, and why it is so.
For the student's explanations, s/he will motion for the student to go up to the chalkboard and

explain his/her answer. If the instructor receives no response, s/he asks a group (rather than an individual) if they know the answer. Throughout these class discussions, the class is extremely quiet.

Observer's note: I think that the students are having some trouble with this section of adding, multiplying, and dividing fractions. They seem to understand how to do the algorithm, but now why it exists. The class seemed quiet throughout the hour, and I think the reason was that the students were uncomfortable with the material and didn't know the answers.

During the first 3 of the 6 problems, only 1/4 students appear to be paying attention to the discussion; the others have blank stares on their faces, or looking down in their workbooks or notebooks.

At 1:16, The instructor brings up division of fractions saying,"We all know how to divide fractions, but why do we do it this way? Can anyone explain why?" One woman starts to explain, and the instructor motions for her to go to the board. The woman says, half-kiddingly, "If I had known I would have to go to the board, I wouldn't have said anything." It appears that the student did not answer in the way that the instructor wanted, so the instructor tells the students discuss for five minutes with their group why (1/2) / (1/3) = 3/2 using pictures. After about a minute, a woman says she can do it, but with rectangles. The instructor says "Fine, do it at the board." The volume of the class is fairly high, and the instructor must raise his/her voice above it to tell the students that this woman has the answer. This woman begins and the instructor helps her by writing something down on her picture; apparently, this confuses the woman and she doesn't know how to finish. A woman from group 1, jumps up to explain it on the black board. Again, this explanation is not what the instructor wants and s/he tells them to discuss the "why" again for another five minutes. After this, the same woman from group 1 says she has figured it out and walks up to the board. She explains it correctly; the instructor smiles indicating she has a correct explanation. A woman near me asks the student to explain it over again, but is not heard by the woman at the board, who sits down. The woman in front of me in group 3 says, "No one ever told me this."

The instructor then writes a more complicated division equation (1/5) / (3/2) = 4/9 on the board and asks the students to solve that with pictures. A woman asks him/her, "Why does the reciprocal work then?" and the instructor responds, "That is what we're trying to figure out." The class is completely quiet while the students work independently on this. Meanwhile, the instructor leans against the table at the right front of the room and surveys the classroom. After three minutes of this extremely quiet classroom, the instructor tells them, jokingly, they can "actually talk" and in reaction, people discuss the problem. After a few minutes, the instructor asks the class, "What is different about this problem [than the earlier one]?" to which one student responds with an answer. After a few minutes, one woman walks up to the board and draws a picture and explains the problem. A woman asks her to repeat it and so she does. The instructor smiles at her explanation and asks if everyone understands. One woman says no. It appears that the instructor does not hear her, and tells the students to work individually on another problem of the same type, to make sure everyone understands. After five minutes, the instructor asks if anyone

has the answer. About three students request two more minutes and so the instructor says okay. After two minutes have passed, The instructor tells a woman from group 3, who appeared to understand it, to go to the board and explain. The woman appears confident with her answer and begins to explain. When she is finished, the instructor asks her if she thinks she is right. and says, "This is very interesting." The woman at the board responds that then it must be wrong and sits down. A woman volunteers that it is wrong in only one part and explains why. Another woman in group 7 asks, "Why do we need to find a common denominator? I used a different method." The instructor asks her to explain by going to the board. The woman says she doesn't want to go to the board because she doesn't know if it is right, but after a little coaxing, goes to the board and explains. The class is completely quiet while she explains, and all appear to be listening as noted by all people are facing the board where she is standing. The instructor asks her if this would work with different numbers; the woman does not know and the instructor leaves this open and moves to the next topic.

The instructor says to the class, "[Dividing by fractions] is hard to understand, but you will have to explain how to divide fractions when you teach." She says that it is no longer acceptable to just explain how, the students must explain why as well.

Finally, a woman from group 1 goes up to the board and explains (correctly) the solution. The entire class is looking at the board and appears to be listening. A woman in group 6 asks her to explain it again, and the woman says, "I didn't do it well the first time. Let me explain it again." After her explanation, this woman says where she teaches, they use a hamburger, lettuce, tomato for fractions: the hamburger is the "pie", the lettuce stands for "1/8," and the tomato for "1/4." The students seem to appreciate this information and nod in approval. A woman in group 1 asks what the answer will be if the division is reversed. The instructor explains this.

At 1:55, the topic then moves on to addition of fractions. The instructor asks the students to show how to add 2/3 and 1/5 by drawing out pictures. The class again is completely quiet and works on it for about a minute. A woman from group 8, says, "This is the one thing I do understand, so I'll do this" and walks up to the board and explains it correctly.

At 2:00, the instructor tells them to work in groups on a new problem. They work in groups until the end of class. During this time, the instructor walks around the groups and asks the members how they are doing. Many times, students will notice him/her walking by and will ask him/her a question. A few times, three students walked around to talk with other groups, for an explanation or to socialize. The groups were situated quite close together and a few times, group members from different groups conversed socially or about the problems. Many of the students get ready to leave at 2:13, and most are gone by 2:15. I overhear some students talking about their weekly Monday meeting: who will go, where and what time. When I leave at 2:17, two students remain, apparently with the intention to do some work in the class.

Group Observations:

Group 1:
There was one person in this group who, every time I looked at the group, was discussing a problem with some other person in her group. This person was very active in both the group and class discussion. This group appeared to discuss ideas as a group, and as a whole there was very little working independently.

Group 2:
This group consisted of an older woman, a male, and a young woman. The older woman appeared to do the majority, if not all, of the explaining, with some input by the other female. When this woman is not explaining, the group appears to work independently. I should note that the other two group members rarely talked to one another, but instead talked to the older woman.

Group 3:
The two women on the "ends" of this group appear to discuss the problems almost continuously during the group work. The other two either work independently, or listen to the discussion, adding input occasionally. At the end of the hour, this is the only group that is remaining a bit to discuss the problem.

Group 4:
This group appears to work independently throughout the group work times. Occasionally, two of the group members will discuss some problem. One member spent some time socializing with a member of group 5.

Group 5:
This group appears to work independently throughout the group work time. One woman talks to a woman in group 4 and at another time a woman in group 3, socially or about the problem.

Group 6:
At the start of the hour, this group socializes until the instructor walks by and says, "Done?". They then don't respond and three of the four begin working; the one who is not working is eating some chips and looking around the classroom. After a few minutes, they all socialize again. Throughout the hour, the earlier woman appeared to be confused as to what they should be doing. Several times, I overheard her say,"What are we supposed to do?" At the end discussion, this group does not socialize and works independently, with some discussion between two of the members.

Group 7:
During the short discussions in the middle of the hour, this group either worked independently, or did not do any work and looked around the classroom.

Group 8:
This group was very near to where the instructor stood for the majority of the hour, and took advantage of his/her close proximity. Throughout the hour, I noticed the instructor talking with this group during all of the five minute discussions. The older woman in particular appeared to

ask the instructor questions throughout the hour.